Alternate Interpolators for rBURS without Deapodization

P. E. Larson[1], J. M. Pauly[1]

[1]Electrical Engineering, Stanford University, Stanford, CA, United States

Introduction:

The Regularized Block Uniform ReSampling (rBURS) algorithm [1-2] is an efficient method for resampling nonuniform samples onto a uniform grid, making it a useful image reconstruction algorithm for k-space trajectories that do not sample on grid points. Neither a 2X grid nor density compensation is required in rBURS, making it faster than gridding when the required inverse matrix is precomputed. The sinc interpolator suggested by Rosenfeld [2] becomes less effective as the block size decreases, most notably causing apodization at the image edges. While deapoidization can be applied, the signal processing technique of windowing can also be used to reduce apodization and aliasing to produce more accurate images.

Theory/Methods:

For every given grid point, the grid points within a radius Δk and nonuniform sample within a radius δk are used for resampling in rBURS. A desirable interpolator should be close to zero beyond δk so energy in the interpolator is not neglected. Additionally, δk should be chosen to be as small as possible such that fewer matrix coefficients are required for reconstruction and speed is increased. For small δk, the sinc interpolator will have significant energy beyond this radius resulting in passband ripple and aliasing. Deapodization can correct for some of this artifact.

The radial nature of the algorithm in selecting grid points and nonuniform samples lends itself to the use of a radial kernel. For a circularly symmetric passband, the optimal interpolator is the $FT\{circ(r)\} = J1(\pi q)/2q = jinc(q)$ where $circ(r) = 1$ for $r < ½$, and zero everywhere else. J_1 is a first order Bessel function of the first kind. The circularly symmetric function *jinc(r)* has a uniform circular response that *sinc(r)* does not have.

Windowing the sinc or jinc interpolator smoothly tapers the function to a finite width, creating a smoother passband and reducing side-lobe levels. The two windows used are: 1) the Hamming window which has a very simple implementation and small transition bandwidth, and 2) the Kaiser window (similar to the Kaiser-Bessel function often used in gridding [3]) which has maximal energy in the region of interest and excellent sidelobe suppression. One drawback is that the windowing operation reduces the passband width of the interpolator, requiring the use of a larger grid.

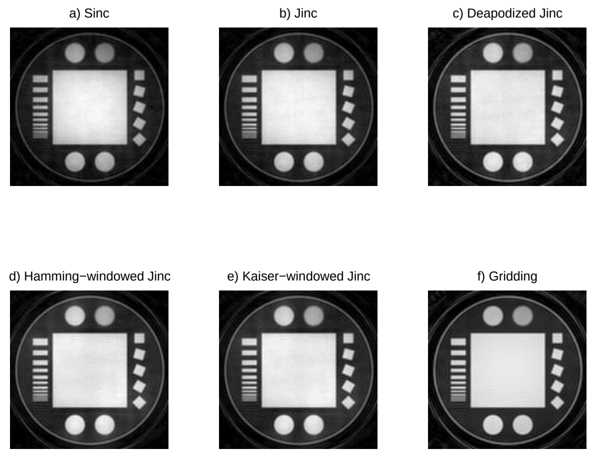

Figure 1: Reconstructed phantom images taken on a GE Signa 1.5T scanner using 6 spiral interleaves.

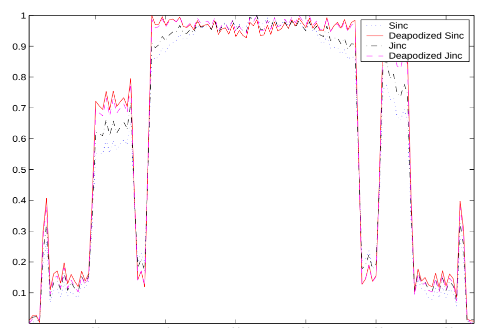

Figure 2: Cross Sections of the Sinc and Jinc interpolators, along with their deapodized recons.

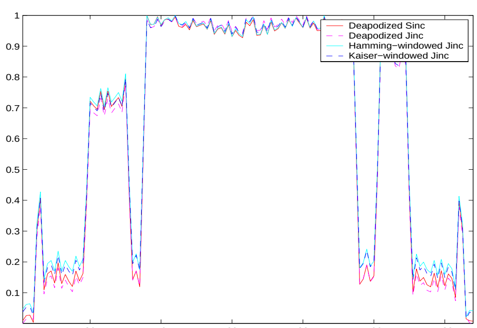

Figure 3: Cross Sections of the deapodized recons, compared the to windowed interpolators.

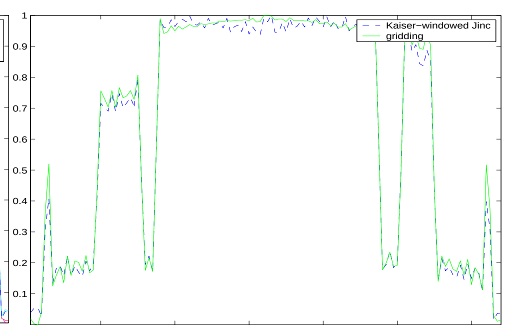

Figure 4: Cross Sections comparing the Kaiser-windowed Jinc interpolator to gridding.

Results/ Discussion:

The reconstruction of a spiral data set with 6 interleaves and 1536 samples per interleave is shown in figure 1. Figures 1a-e used various rBURS interpolators with parameters $\delta k = 1.25$, $\Delta k = 2.5$, and $\rho = 0.3$ (regularization parameter, chosen by iteration) onto a 192x192 grid, with only the 128x128 region of interest displayed. The windowed interpolators are zero beyond a radius of Δk. 45190 matrix coefficients/multiplications were required for the resampling, compared to 45618 with the same parameters on a 128x128 grid. Figure 1f was reconstructed using gridding on a 256x256 grid with a window width of 2.5 [3].

Figures 2-4 shows cross sections through the middle row of reconstructed images using the same data set and parameters as figure 1. In figure 2, the sinc and jinc interpolators are compared along with their deapodizations. On the 1.5X grid, the jinc performs slightly better, but this is not necessarily the case if a 1X grid is used. Since the windowed interpolators require a 1.5X grid, windowing the jinc function will give better results. When testing windowed sinc functions, sometimes small regions towards the corners of the image would lose signal, which is corrected by using a windowed jinc function.

Figure 3 demonstrates that the windowed jinc functions perform comparably to the deapodized reconstructions. They are nearly identical, except the windowed functions have slightly less apodization near the image edges. A small amount of undesirable rippling is present in the center portion.

Figure 4 compares the Kaiser-windowed jinc function to the gridding reconstruction. The results are very similar, although gridding still has a slightly flatter response. This could be due to the required regularization or the width of the windowed interpolator, but the difference is very small.

Conclusions:

Using windowed jinc functions as interpolators in rBURS allows for fast and accurate image reconstruction from nonuniform samples without the need for deapodization. The jinc function provides a circularly flat response and behaves better than the sinc function when windowed. A slightly larger resampling grid size is required, where a 1.5X grid was used in this abstract and is found to be adequate. The Kaiser-windowed jinc function performs slightly better than the Hamming-windowed jinc function. Other window functions can be used but they will also give very similar results.

References:

[1] Rosenfeld D, *MRM* 40: 14-23 (1998). [2] Rosenfeld D, *MRM* 48:193-202 (2002). [3] Jackson JI, *et al*, *IEEE T-MI* 10: 473-478 (1991).