Analytical Investigation of Bias in Array Coil Combination

M. Bydder[1]
[1]Radiology, UCSD, San Diego, CA, United States

The individual images obtained from an *n*-element phased array coil can be combined into a single image in such a way as to maximise the signal-to-noise ratio (SNR) using Summation Using Profiles Estimated by Ratios (SUPER) (1). The SUPER technique assembles the individual signals s_k from each coil into a quotient Q

$$Q = \sum_k s_k \langle s_k \rangle^* \Big/ \sum_k \langle s_k \rangle^2 \qquad [1]$$

where $\langle\ \rangle$ and * denote smoothing and complex conjugation, respectively, from which are obtained magnitude and real SUPER

$$M = \sqrt{\sum_k \langle s_k \rangle^2}\ |Q| \qquad \text{and} \qquad R = \sqrt{\sum_k \langle s_k \rangle^2}\ \mathrm{Re}\{Q\} \qquad [2]$$

These methods reduce bias in the image compared with the sum-of-squares S, as illustrated in the following expressions: $S = \sqrt{T^2 + 2n\sigma^2}$, $M = \sqrt{T^2 + 2\sigma^2}$ and $R = \sqrt{T^2 + \sigma^2}$, where T is the true underlying signal and σ^2 is the noise variance on the real and imaginary signals from each coil. These were derived following similar reasoning to (1).

In practice the degree of bias reduction in *M* and *R* depends on performing optimal smoothing such that (**i**) noise is eliminated and (**ii**) the coil sensitivity profiles are not distorted. This basically amounts to finding a parameter μ for the smoothing method; an empirical approach is make magnitude SUPER images M_μ^2 for various choices of μ and plot

$$\overline{\beta}_\mu = \langle S^2 - M_\mu^2 \rangle / 2 \qquad [3]$$

against μ to locate the point where over-smoothing introduces defects. In Eq [3], $\langle\ \rangle$ represents full field of view averaging. For low-pass filtering with a 2-dimensional uniform convolution kernel of side μ (ie. moving average), the expression can be evaluated numerically to conform to the following relationship

$$\overline{\beta}_\mu = \left(1 - \mu^{-2}\right)\overline{\beta} \qquad [4]$$

where $\overline{\beta} = (n-1)\sigma^2$ is the optimal as given in (1), although a derivation to prove this analytically has not been possible. Plotting $\overline{\beta}_\mu$ against $1-\mu^{-2}$ gives a straight line with zero intercept and gradient = $\overline{\beta}$ (see Fig 1) from which σ^2 can be obtained. This provides a way of determining the noise variance in an image using a single acquisition without user intervention (cf. ref 2).

Eq [4] indicates that the optimal bias reduction is only obtained for infinite smoothing $(\mu \to \infty)$, which is clearly unfeasible without violating condition (ii). So instead, Eq [4] may be used to assess the case of less than optimal smoothing – for example, with μ = 5 one may see that $(1-\mu^{-2}) \geq 0.95$ so the amount of bias reduction is better than 95% of the optimal. This degree of smoothing is far milder than the 32×32 Hanning window used in (1), roughly comparable to μ = 13, so the risk of violating (ii) is greatly reduced for only a small loss of performance. With real SUPER, the equivalent of Eq 4 is $\overline{\beta}_\mu = (1-\mu^{-2})(n-\tfrac{1}{2})\sigma^2$ and so it can been seen that real SUPER outperforms optimal magnitude SUPER in bias reduction whenever $\mu^2 \geq 2n-1$.

Bias Correction

Rearranging the expressions for *S* and *M* above and eliminating σ^2 gives an estimate for *T* that is in principle completely free of bias; it is referred to as unbiased SUPER, *U*.

$$U = \sqrt{S^2 - n\langle S^2 - M^2 \rangle / (n-1)} \qquad [5]$$

This procedure works by subtracting out the known quantity of bias in *S* to leave just the true signal. The performance of *U* is comparable to that of *R* and is achieved without assuming slowly varying image phase, as required by the latter method.

Conclusion

The amount of bias in array coil combination is a simple function of the parameter value used. It can also be directly measured and used to determine properties about the noise in the image.

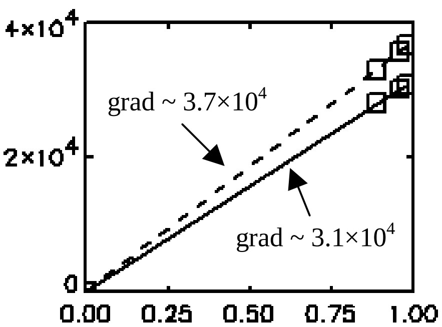

Figure 1 Plot of $\overline{\beta}_\mu$ against smoothing parameter $1-\mu^{-2}$ for magnitude SUPER (solid line) and real SUPER (dashed line) of an n = 4 coil phantom data set. The gradient is expected to be $(n-1)\sigma^2$ and $(n-\tfrac{1}{2})\sigma^2$.

References
(1) Bydder M et al. *Magn Reson Med* 2002;**47**:539-48
(2) Firbank MJ et al. *Phys Med Biol* 1999;**44**:N261–N264