Fast Gridding Methods for 3D Reconstruction

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Introduction Reconstructing high-resolution 3D images from non-uniformly sampled k-space data can require gigabytes of memory and take hours to reconstruct even on modern computers. Gridding, where data is convolved with a gridding kernel and the result sampled onto a Cartesian grid, is an efficient reconstruction technique. Gridding has a number of parameters and variations which allow for great flexibility but which must be chosen well to produce accurate reconstructions in minimum time. We show a gridding method which is particularly well suited to 3D reconstruction, producing acceptable error levels while reducing memory requirements about 4X with comparable improvements in speed. Gridding incurs reconstruction errors due to aliasing of the Fourier transform of the kernel. Jackson *et al.* reduce these errors to an acceptable level by doubling the image field-of-view (2X grid) [1]. In 3D this results in an often unacceptable 8X increase in image size. We propose using a smaller image field-of-view such as a 1.25X grid, only increasing image size 2X.

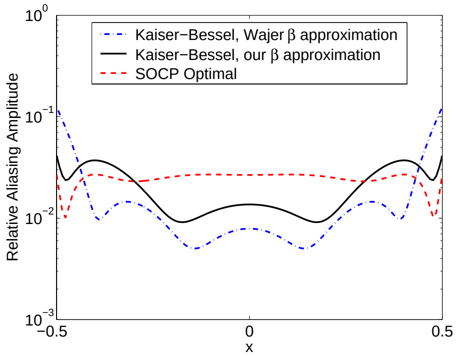

Figure 1 *Aliasing amplitude for α=1.25, αW=3. Maximum of 0.125 (Wajer β), 0.042 (our β), 0.027 (SOCP).*

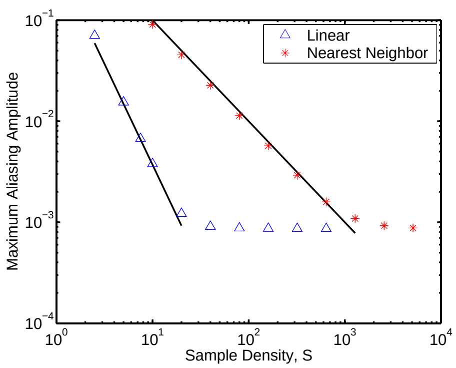

Figure 2 *Decrease in aliasing as kernel sample density is increased, α=1.25, αW=6.*

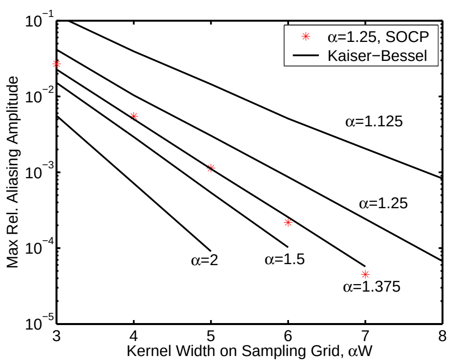

Figure 3 *Maximum aliasing amplitude of Kaiser-Bessel function vs. kernel width for a selection of grid sizes. Also, the SOCP optimal result is shown for α=1.25.*

Theory Central to achieving acceptable results on a 1.25X grid is choosing an appropriate kernel. Jackson *et al.* has shown that a Kaiser-Bessel function with a well chosen shape parameter, β, is near-optimal [1]. In general, β should be chosen such that the main lobe of the Fourier transform of the kernel does not alias back into the image. This is achieved when $\beta = \pi\sqrt{W^2(\alpha - 1/2)^2 - u}$, where W is the width of the kernel on a 1X grid, α is the grid size (αX grid) and u is a value between 0 and 1 which depends on how we define the width of the main lobe. Unfortunately, repeated evaluation of the Kaiser-Bessel function in gridding is very costly. By sampling the kernel, an evaluation becomes a simple lookup operation which further accelerates reconstruction.

Jackson measures the performance of the gridding operation as the relative amount of aliasing energy over the entire image [1]. In MR, we are more concerned with the amplitude of the aliasing artifact which is the square root of the aliasing energy. We further extend this measure as the relative amount of aliasing amplitude at each location in the image. By ensuring that the maximum of this measure is below some value, such as the SNR, we can ensure that visible gridding artifacts will not appear in the image. Choosing the kernel samples which minimize the maximum aliasing amplitude is an optimization problem which can be solved using second-order cone programming (SOCP) [2].

Methods and Results We derived a closed form solution for the aliasing amplitude as a function of the number of kernel samples. Using this, we can quickly plot aliasing amplitude as a function of position as shown in Figure 1. We found that the maximum aliasing amplitude for the Kaiser-Bessel function is minimized when $u\approx0.8$. The expression used by Wajer *et al.* corresponds to u=0, which works well for larger grids such as 2X, but can cause aliasing artifacts at the edge of the image in smaller grid sizes, as shown in Fig. 1 [3].

Our solution also predicts that the aliasing amplitude due to sampling the kernel has a maximum at the edge of the image equal to about $1/S$ for nearest neighbor interpolation and $0.37/S^2$ for linear interpolation, where S is the number of samples per unit on a 1X grid. This agrees very well with our calculated results shown in Fig. 2.

We implemented an SOCP to minimize the maximum aliasing amplitude. Figure 3 shows that using this kernel on a 1.25X grid achieves similar performance to the best Kaiser-Bessel function on a 1.375X grid.

Discussion A design example illustrates the power of our method. Suppose we wish to reconstruct an image with an SNR of 40 (relative noise level 1/40=0.025). Desiring our aliasing energy to be an order of magnitude below the noise, we might choose to sample a Kaiser-Bessel function with an αW (the kernel width on the sampling grid) of 6 on a 1.25X grid. Figure 3 shows an aliasing amplitude of about 10^{-3} for this choice. To ensure that our sampling does not cause additional error, we choose the maximum sampling aliasing amplitude to be 10^{-4}. For nearest neighbor kernel interpolation, S = 10,000 requiring us to sample the kernel with WS=48,000 points. The same results can be achieved with linear kernel interpolation and sampling with only 300 points. Linear interpolation requires an extra lookup and multiplication, but offers a dramatic decrease in the samples we must store. Table 1 shows the dramatic results that can be obtained for a 128^3 image.

Gridding Method	Memory	Time
2X grid, cone kernel	128 MB	37 sec.
1.25X grid, Kaiser-Bessel kernel	32 MB	30 min.
Our method	32 MB	6 sec.

Table 1 *Typical memory and time requirements for reconstruction of a 128x128x128 image. Calculated on a 2GHz PC with 512 MB memory.*

Conclusion Using a smaller grid size, such as 1.25X, and an appropriately sampled kernel we can achieve accurate reconstruction while reducing our memory requirement 4X and significantly decreasing reconstruction time, both of critical importance in 3D image reconstruction.

References

1. Jackson, J.I., *et al.*, IEEE Trans. Med. Imag., 10, 473-478, 1991.
2. Lobo M., *et al.* Linear Algebra Appl. 284: 193-228, 1998.
3. Wajer, F.T.A.W. *et al.*, Proc. ISMRM, 7th Ann. Mtg., 663, 1999