Regridding Using Matrix Methods To Reduce Artefacts

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Introduction

Regridding k-space samples acquired with irregular spacing is a well-known issue in MRI. The task can be posed as an inverse problem of finding the values on a Cartesian grid (**x**) that when operated on by a matrix of interpolation coefficients (**A**) give the best approximation to the acquired k-space samples (**b**) (1):

$$\mathbf{A}\,\mathbf{x} = \mathbf{b} \qquad [1]$$

The matrix **A** is composed of sinc terms and has m rows and n columns with m and n of the order 10^5, thus a direct solution can be time-consuming (2). The considerable literature on gridding is largely an attempt to solve Eq 1 by different means. The initial step is usually to adopt a tapered sinc or Gaussian-like kernel to limit the region of support for the interpolation, which makes **A** into a sparse matrix and reduces storage and computation demands. Some error is introduced from doing this although the error can be reduced to a negligible level with a suitable interpolation kernel even with a region of support containing $r << n$ points (3).

Conventional gridding methods approximate the pseudo-inverse $\mathbf{A}^{-1}$ as a product $\mathbf{A}^T\mathbf{D}$, where **D** is a diagonal (density compensation) matrix resembling $(\mathbf{A}\mathbf{A}^T)^{-1}$ (4). Figure 1 shows an example of $(\mathbf{A}\mathbf{A}^T)^{-1}$ for a radial data set, as calculated by singular value decomposition, showing many off-diagonal elements. The bandwidth may be increased to improve the approximation, however a different approach is to solve Eq 1 by iterative methods. These calculate **x** to a specified accuracy and are computationally feasible when **A** is a sparse matrix. Another advantage of using the matrix approach is that it can make use of existing numerical techniques for solving linear systems, eg. conjugate gradient which minimises the least squares error between **b** and **Ax**. In this abstract Lp regression and orthogonal Lp regression are considered (5), which are methods for dealing with gross errors in **b** and **A**:

$$\min \| \mathbf{b} - \mathbf{A}\mathbf{x} \|_p \qquad [2]$$

$$\min \| \mathbf{Z}\mathbf{y} \|_p \qquad [3]$$

where $\| \cdot \|_p$ is a p-norm, $\mathbf{Z} = [\mathbf{A},-\mathbf{b}]$, $\mathbf{y} = [\mathbf{x},1]^T$ and $\mathbf{y}^T\mathbf{y} = 1$. In the context of MR imaging, gross errors may occur as a result of subject motion and hardware imperfections.

Methods

A sparse conjugate gradient solver was implemented with Lagrangian interpolation coefficients. Bilinear (r=4), bicubic (r=16) and biqunitic (r=36) were used for their simplicity and the bicubic kernel selected as a reasonable compromise between accuracy and reconstruction time (several seconds on a 733 MHz P3). Data were acquired using a radial k-space trajectory as described in (6). For the purposes of testing the reconstruction, the data were corrupted with a single spike value.

Results

Figure 2 shows the image reconstructed by Eq 1 (least squares) and Figure 3 shows the image reconstructed by Eq 2 with p = 1. The spike artifact has been completely removed from the data, however there are other artefacts in the image. These could be due to the different delays on each gradient during ramp-up, which result in errors at the center of k-space due to inaccuracies in the k-space trajectory. The reduction of these errors using Eq 3 is the subject of on-going work.

Figure 1 The matrix $(\mathbf{A}\mathbf{A}^T)^{-1}$

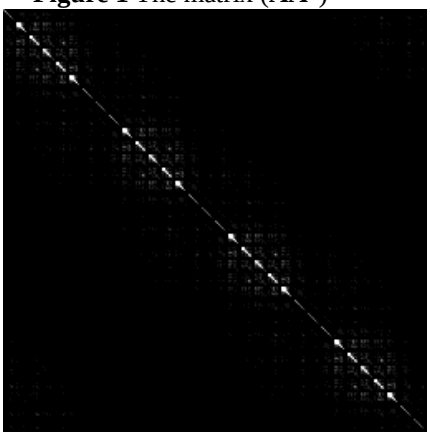


Figure 2 Reconstruction with p = 2

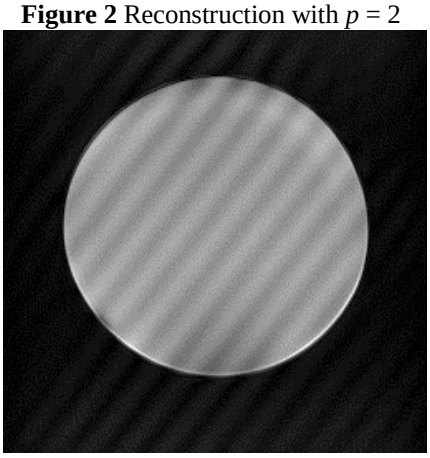
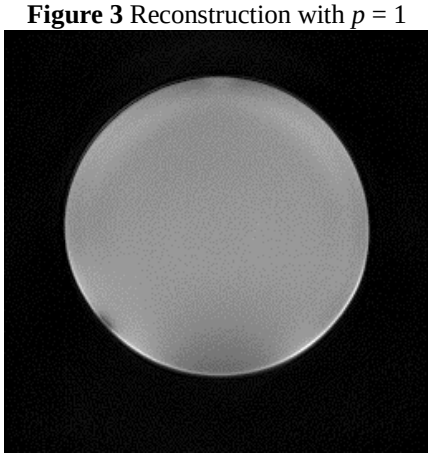


Figure 3 Reconstruction with p = 1



Discussion

Increases in processor speeds mean that inverse problems such as Eq 1 can be solved using fewer approximations and with the ability to reduce errors by using more computationally expensive techniques than previously. Many sources of error in non-Cartesian imaging, such as finite kernel size, subject motion and hardware imperfection, can be reduced in this way. Similar methods have been used in parallel imaging reconstruction (7,8).

References

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