Gradient Waveform Design Using Genetic Programming

D. R. Thedens[1], M. Navalgund[1]

[1]Electrical and Computer Engineering, University of Iowa, Iowa City, IA, United States

Introduction: While two- and three-dimensional Fourier Transform imaging remain the predominant image acquisition methods, more sophisticated *k*-space sampling patterns such as spiral and Lissajous trajectories have found increasing application. Furthermore, interest in variable density sampling trajectories has also arisen to improve contrast, correct for motion, minimize imaging time, and tailor the sampling to the object being imaged. The implementation of such trajectories may require considerable numerical analysis to generate efficient gradient waveforms that still meet hardware constraints on gradient strength and slew rate. The process must be repeated for each change in image acquisition parameters, as closed form solutions are rarely available.

In this work, we investigate the design of gradient waveforms for arbitrary sampling trajectories using genetic programming (GP). GP belongs to a class of optimization algorithms that mimic the process of evolution. These algorithms create an ever-improving set (*population*) of feasible solutions by combining properties of existing solutions (*crossover*) and randomly making small alterations in solutions (*mutation*), preferentially choosing the better solutions from the population for these operations based on a figure of merit or *fitness*. The unique element of GP is that the solution space consists of mathematical functions or computer programs rather than vectors of parameters. The solutions represent the gradient waveform as a continuous function of time and possibly other parameters that can be used to generate families of *k*-space trajectories. The purpose of this work was to evaluate the feasibility of the GP approach on the design of variable-density spiral gradient waveforms.

Methods: Gradient waveforms for spiral sampling in two dimensions are described most generally as parametric equations with $k_x(t) = r(\theta(t)) \cos(\theta(t))$ and $k_y(t) = r(\theta(t)) \sin(\theta(t))$. $\theta(t)$ describes the angular location (and velocity) of the trajectory and $r(\theta(t))$ relates the radius of the points to the angle, setting the density of the spiral samples. In this GP formulation, $r(\theta(t))$ is chosen for the desired density ($r(\theta(t)) = \theta(t)$ for uniform density), and the GP algorithm evolves a solution for $\theta(t)$ as a continuous function of time. These functions are represented as binary trees as demonstrated in Figure 1. The fitness of an individual solution $\theta(t)$ uses the gradient and slew rate magnitude at many sampled points along the gradient waveforms described by $\theta(t)$, which requires the calculation of derivatives of $\theta(t)$ and $r(\theta(t))$. The fitness is computed as a weighted sum of the squared difference of the actual gradient magnitude and the optimum (maximum) value, along with non-linear penalty terms to enforce gradient and slew rate limits.

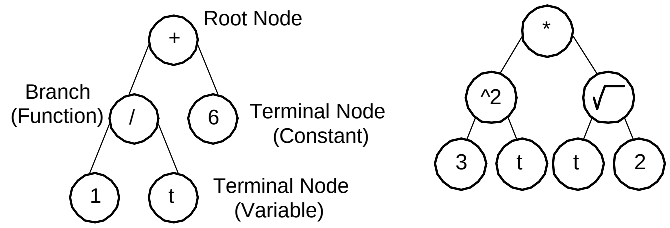

Figure 1: Representation of functions in the solution space. The functions are stored as binary trees with binary functions at the branches and constants or independent variables at the leaves. Unary functions use the absolute value of the sum of the branches as their argument. The tree at left represents $f(t) = (1/t) + 6$, while the tree on the right is $f(t) = (3+t)^2(|t+2|)^{1/2}$.

The GP algorithm is initialized by generating a random population of trees using binary functions from the set $\{x+y, x-y, x*y, x/y, (|x+y|)^{1/2}, (x+y)^{1/4}\}$ along with terminal values $\{1,2,3,4,5,t\}$ where t is the independent variable. Other functions and constants may be evolved within the tree representation during the course of the algorithm. For each iteration or *generation*, the fitness of each individual $\theta(t)$ from the population is computed and the population is ranked. Random pairs of individuals are selected such that higher fitness individuals are more likely to be chosen, and the crossover operation that "blends" the two trees into two new individuals is performed by selecting a random point in each tree and swapping the parts below. Individuals are similarly chosen for mutation by random perturbations in the tree. The resulting trees form the new generation and will have solutions with both higher and lower fitness values, with higher fitness individuals being more likely to be retained into succeeding generations. The process continues until convergence to the highest fitness value or (more often) until a maximum number of generations have been created. The individual tree representing $\theta(t)$ with the highest fitness is chosen as the optimal spiral gradient waveform.

Results: After considerable experimentation with population sizes and selection methods, the best results were achieved with a population size of 500 individuals, a mixture of selection criteria based on absolute fitness value and rank in the population, and termination after a maximum of 150 generations. Smaller populations and selection based only on absolute fitness values yielded populations that converged to sets of identical individuals before achieving a good solution, and solutions rarely improved after 150 generations. The figure below shows the gradient and slew rate magnitudes for the optimal $\theta(t)$ found for a non-uniform density spiral calculated over a 100 ms window. The gradient is well matched to the maximum amplitude of 22 mT/m used here. In the slew rate limited portion of the waveform, the actual values vary somewhat, partly due to the sampling of the waveforms to generate the fitness and to termination of the algorithm before reaching the true optimum.

Discussion: We have designed and implemented a GP optimization method for generating spiral gradient waveforms in the form of continuous functions. The advantage to a functional optimization is that the resulting solutions can be arbitrarily resampled and scaled in time to produce a family of spiral or other acquisitions. This work has demonstrated the feasibility of this approach. While the solutions produced are inevitably complex (as in the 40 terms in the solution for the example below), the representation of the function in binary tree form makes the calculations straightforward. However, this method is sensitive to several parameters such as population size and selection mechanisms, and the probabilistic nature of GP may still yield suboptimal solutions. Future improvements to this method will include determination of a robust set of GP parameters, and incorporation of additional functions and independent variables including imaging parameters such as resolution.

References:

[1] Koza, J. R. *Genetic Programming: On the Programming of Computers by Natural Selection.* MIT Press, Cambridge, MA, 1992.

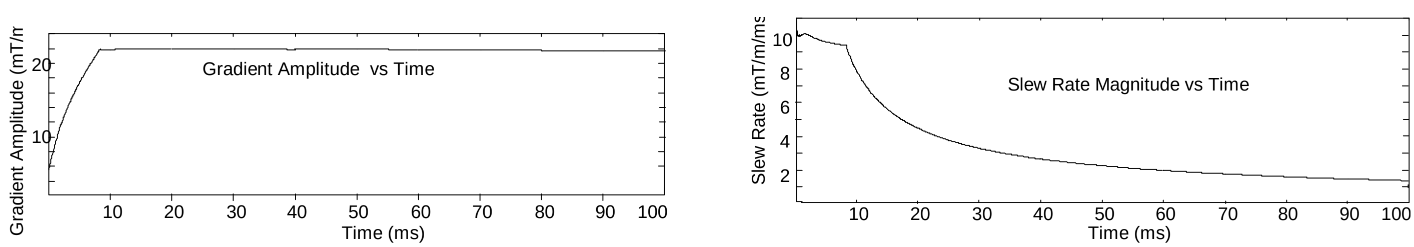

Figure 2: Sample gradient characteristics for a variable density spiral acquisition. The waveforms are derived from the final tree representing the best $\theta(t)$ found. The sampled locations for fitness did not include the *k*-space origin, which results in a discontinuity at the origin. The expression for the final $\theta(t)$ included 40 terms.