Optimal Readout for Fast Imaging in the Presence of Short T_2

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Introduction

We analyze the problem of finding the readout time which maximizes the SNR when performing MRI of short T_2 species. We demonstrate that for a fixed voxel size and a known mono-exponential T_2, the optimal readout time can be accurately estimated using an analytic approximation. We have found that this approximation agrees well with experimental results at that it can be easily generalized when considering more complex relaxation behavior such as in multiple quantum sodium MRI.

Theory

In this work we approach the problem of optimizing the MRI signal acquisition when the T_2 relaxation time is comparable in length to the readout time. More specifically, for a fixed acquisition protocol, when an estimate of the T_2 decay time is available a priori, we are looking for a prescription to choose the readout time, T_{read}, that maximizes the signal-to-noise ratio (SNR) in the reconstructed image. We assume that the time decay of the signal for each point in the image-space is described by $\rho(r,t)$. The form of this function could be as simple as a mono-exponential function or more complex as in the case of multiple quantum-filtered experiments. We also consider that the data required for reconstructing the entire image are acquired by performing N separate acquisitions (RF excitations or shots); the total acquisition time is then given by $T_A = N\,T_{read}$. Assuming that the sensitivity of the RF coil is constant across the FOV, the measurement noise can be described as an additive Gaussian random process with zero mean and variance σ. If the standard regridding inversion algorithm is used [1], the intensity of the signal can be expressed in terms of the gridding kernel, $W(.)$, and the sample weighting factor c, that is: $I(r) = c(r)^{-1}\int_\Omega e^{-2\pi i\kappa r} R(\kappa)\, d^3\kappa$ and where the K-space intensity factor is $R(\kappa) = \int_0^{T_A} dt\, W(\kappa, p(t))\, S_{meas}(t)$. Using these definitions, the reconstruction formula can be expressed as $I(r) = \int_0^{T_A} dt\, \Phi_p(r,t)\, S_{meas}(t)$, where the kernel Φ_p describes the way in which the K-space samples measured at a time t, are spread over the image. The subscript p emphasizes the kernel's functional dependence of the K-space trajectory $p(t)$. Using the expressions above, the variance of the noise and the SNR can be written as:

$$\sigma^2(r) = \sigma^2 \int_0^{T_A} dt \left|\Phi_p(r,t)\right|^2 \quad \text{and} \quad SNR(r) = \sigma(r)^{-1}\sqrt{I_s(r) I_s^*(r)} = \sigma^{-1}\left|\int_0^{T_A} dt\, S(t)\,\Phi_p(r,t)\right| \left(\int_0^{T_A} dt \left|\Phi_p(r,t)\right|^2\right)^{-1/2}$$

An upper bound for the optimal readout can then be obtained from the formula above by considering that the imaged object can be described as a delta function in the center of FOV. In this case, the SNR is given by $SNR = \sqrt{N/T_{read}} \int_0^{T_{read}} dt\, \rho(t)$, and the best readout time satisfies the equation $\int_0^{T_{read}} dt\,\rho(t) = 2T_{read}\,\rho(T_{read})$, which has the solution $T_{read} \cong 1.256\, T_2$ for a simple, mono-exponential decay. If the imaged object is not a delta function, but instead has a width $2a$, the formula above generalizes to

$$SNR(\beta) = e^{-\beta}\sqrt{\beta}\int_{-1}^{1} dy\, e^{-\beta y}\,\mathrm{sinc}(\varphi y) = \frac{i e^{-\beta}\sqrt{\beta}}{2\varphi}\left\{\mathrm{Ei}_2(-\beta - i\varphi) - \mathrm{Ei}_2(-\beta + i\varphi) - \mathrm{Ei}_2(\beta + i\varphi) + \mathrm{Ei}_2(\beta - i\varphi)\right\}, \mathrm{Ei}_2(z) = \mathrm{Ei}(z) - \log(z)$$

where, $\beta = T_{read}/2T_2$ and $\phi = 2\pi a K_{max}$, and $Ei(z)$ is the exponential integral [2].

Methods

Computer simulations as well as experimental data were used to verify the predictions from the model. In the computer simulated data set, Monte Carlo simulations (1024 noise seeds) of a 1D imaging experiment with FOV $=20$ cm, 64 points image pixels, $T_2=16$ ms, and a sampling rate $dt=8$ µs were used to estimate the change in SNR as a function of T_{Read}. The experimental data sets were obtained on a whole-body, 3 Tesla MRI scanner (GEMS) using a custom-built sodium RF coil and a twisted projection imaging sequence [3] to acquire images from a phantom with different readout times. These latter set of experiments also included the experimental determination of the T_2 for the phantom. All measurements were performed using pixels in the center of image to avoid the well-known edge effects observed in gridding reconstructions. Gridding was performed using a Kaiser-Bessel Kernel with a window width of *2*.

Results

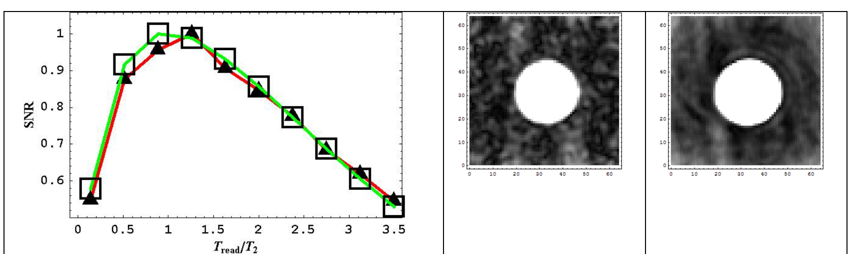

Figure 1: (left) *Comparison of the measured (squares) and theoretically predicted SNR (tringles) for the model described above. Selected partition from 3D sodium images for a homogeneous phantom acquired with redout times of 14ms (middle) and 42ms (right).*

The plot in figure 1 (left) presents a comparison between the theoretically predicted and measured SNR values for the Monte Carlo simulations. The graph clearly demonstrates good agreement between the model and the Monte Carlo data. Similar findings for the experimental data were observed by using the phantom images in figure 1 (T_{Read}=14/42ms, middle and right, respectively) to mesure the change in SNR. The change in SNR (40%) agrees to within 13% with the theoretically predicted values.

Conclusions

The results above demonstrate that readout times play an important role during the optimization of imaging sequences for short-T_2 species and that optimal readout times can be accurately predicted using our model.

References:

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