Accelerated MR Imaging via FOLDing the non-Fourier Encoded Dimensions.

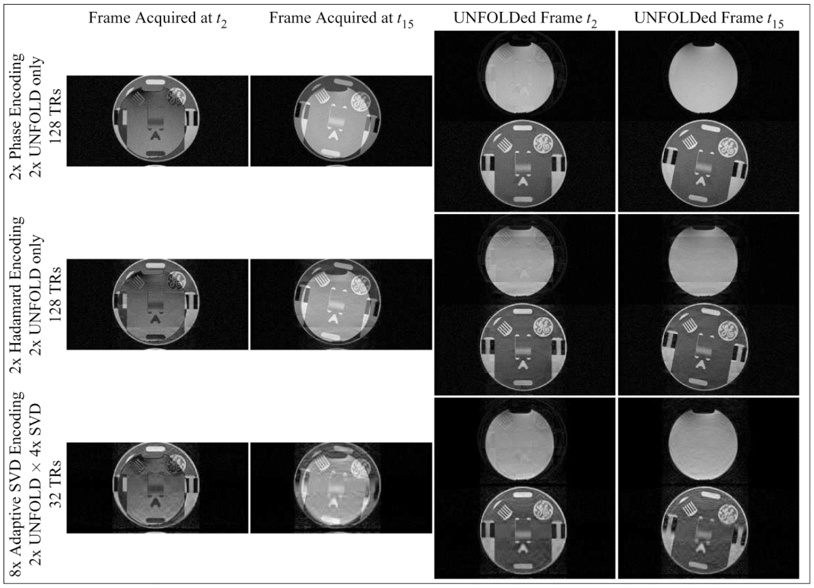
D. Mitsouras[1], B. Madore[2], F. J. Rybicki[2], L. P. Panych[2], A. Edelman[1], G. P. Zientara[2]

[1]EECS, Lab For Computer Science, Massachusetts Institute of Technology, Cambridge, MA, United States, [2]Radiology, Brigham And Women's Hospital, Boston, MA, United States

Introduction: UNaliasing by Fourier-encoding the Overlaps using the temporaL Dimension (UNFOLD) [1] compacts *k*-space (i.e., spatial) information by encoding it using empty portions of the time dimension. The UNFOLD method acquires a sub-sampled *k*-space at each time frame in a series of images so that portions of the FOV are aliased. The overlapped regions of the FOV are resolved by inducing Fourier basis functions over the overlaps in the time dimension (i.e., over the frames). These basis functions are in turn produced via appropriately shifting the phase encode *sampling* function at each *k*-space acquisition time frame. By cycling through these Fourier basis functions in the time domain (i.e., in between images) the aliases are Fourier encoded along the time axis, which the UNFOLD algorithm subsequently decodes. The combination of UNFOLD with non-Fourier encoding relies on two facts; first, just as sub-sampling phase encodes in the acquisition of *k*-space produces aliased overlaps, sub-sampling the *k*-space *excitation* samples also produces aliasing sidelobes of the excitation profile. Second, just as a shift in the acquisition *k*-space sampling function induces a Fourier basis function over the aliases of the FOV, a shift in the *excitation* *k*-space sampling function induces the same basis function over the aliases of the excitation profile. Non-Fourier encoding employs spatial excitations to efficiently encode *k*-space information. UNFOLD non-Fourier encoding (UNFOLDNF) skips excitation *k*-space samples so that copies of the spatial encoding profile are produced over the desired FOV. Thus each overlap of the FOV is encoded simultaneously and identically. Furthermore, the RF encoding scheme (i.e., spatial profiles) of each UNFOLDNF-encoded time frame is changed so that a Fourier basis function is induced over the aliases of the excited spatial profiles. This is accomplished by appropriately shifting the RF excitations under the spatially selective gradient. Once each UNFOLDNF time frame has been non-Fourier encoded, acquired, and reconstructed, the UNFOLD method can be used in its original form to decode the aliased overlaps. With UNFOLDNF, a non-Fourier encoding speedup can be employed in addition to that of UNFOLD since non-Fourier encoding allows one to compact the acquisition of a *k*-space [2] into its statistically significant (w.r.t. noise) components only [5]. That is, a speedup is obtained in the acquisition of the sub-sampled *k*-space matrices that UNFOLD expects, by using fewer spatial encoding functions than number of phase encodes necessary to achieve the desired spatial resolution.

Theory: Using RF encoding, the MR image acquisition process can be modeled as a linear system, $y=xF$, where x and y are row vectors representing the input RF and acquired data, respectively, and F is a system response matrix (related to the *k*-space matrix) [4]. The spatially selective gradient used with the RF excitation x, determines the FOV. Fourier-based acquisitions encode this matrix directly. UNFOLD achieves a reduction in acquisition time by acquiring a limited number of rows of F at each time frame, while shifting which set of lines is acquired at each time frame. This is modeled using a "sub-sampling" matrix D_l that contains a single 1 in each row, with each row representing one encoding step (i.e., sequence repetition). The 1 in each row corresponds to the line of *k*-space that is acquired at that encoding step. At each time frame $l=1,\ldots,L$, UNFOLD uses a different matrix D_l. In order to induce the Fourier basis functions on the overlaps, in MATLAB-style notation, $D_l = eye(mod(l\text{-}1, f)+1{:}f{:}M,:)$, where *eye* is the M-by-M identity matrix, if the resolution along the phase encode dimension is M and an UNFOLD speedup factor of f is employed. To enable non-Fourier encoding with UNFOLD, we note that at each time frame l, the UNFOLD algorithm expects the *k*-space matrix $D_l F$. Thus, at each time frame, non-Fourier encoding needs to encode and reconstruct that matrix. Given the periodicity of the D_l matrices (of f), we can build a composite system response by concatenating the sub-sampled *k*-space matrices: $F_{UNF} = [\ D_1 F, \ldots, D_f F\ ]$. This composite system response matrix can now be non-Fourier encoded as before: $y_{UNF} = xF_{UNF}$. That is, the set of RF encoding functions used (x) must form a basis that is capable of encoding all of the sub-sampled system responses simultaneously. Consequently, both the number of elements of each x is reduced, and the number of encoding vectors needed to fully encode the composite system response is reduced (since the dimensionality of F_{UNF} is smaller by a factor of f). This system response reveals, however, that it is the excitation of each x that must be changed between time frames (i.e., induce the encoding of the aliases). In order for the vector x to encode $D_l F$ at time frame l, one must use the excitation $x_l = xD_l$. It is the vector x_l that, when applied under the same spatially selective gradient, produces the response $y=xD_l F$. This shift of the RF encoding vector x at each time frame is simple to accomplish (e.g., by use of the rephasing lobe). This framework allows the use of any set of encoding functions x, e.g., Hadamard, Wavelet, etc., or any set described by an invertible matrix X. The encoding matrix is at most M/f-by-M/f, reflecting the achieved efficiency. For an SVD-based acquisition [2], the input RF vectors are formed from the dominant left singular vectors of F_{UNF}. That is, if $F_{UNF} := U\Sigma V^H$ with U,V unitary and orthogonal and Σ real and diagonal, the collection of input row vectors X is chosen from the top rows of U^H. Using these, a minimum square-error estimate of the composite system response can be formed via $\hat{F}_{UNF} = X^H Y_{UNF} = \{X^H X\} F_{UNF}$. After all L/f such system responses have been acquired and reconstructed (so as to produce L time frames), UNFOLD can be performed on the L/f collection of $\hat{F}_{UNF}$ composite responses to reconstruct the series of images.

Experimental Results: Data was acquired using a 1.5T MR scanner (Signa LX EchoSpeed, GE Medical Systems, Milwaukee, WI). An UNFOLD speedup factor of 2 was used for all experiments. A non-Fourier encoded spin-echo pulse sequence [3] was used to excite the half-length, appropriately shifted (at each time frame), shaped RF encoding pulses and acquire the system response at each time frame. Reconstruction, by first inverting the non-Fourier encoding process to produce the $D_l F$ of each time frame, and subsequently using UNFOLD on these, produced the results in the figure. The encoding functions used for the Adaptive SVD UNFOLDNF results were computed on-the-fly, adapting them to the contents of the FOV at the frequency of the UNFOLD speedup (i.e., every 2 time frames) as described in the theory section. The standard phase encoded (vertical axis) and non-Fourier Hadamard and Adaptive SVD encoded (also along vertical axis) results are presented in the figure. The lower doped water phantom was rotated about 15 deg. clockwise over the course of the time frames.



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