# A Nonparametric Method for Estimation of Arterial Wall Shear Stress

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## INTRODUCTION

Arterial wall shear stress (WSS) is proportional to the derivative of blood velocity evaluated at the arterial wall. One method for noninvasively estimating WSS is through post-processing of phase contrast magnetic resonance images (PC-MRI). PC-MRI is capable of measuring blood velocity along the direction of arterial blood flow [1]. Parametric methods for estimating WSS from PC-MRI depend on assumptions such as approximately circular vessel symmetry or laminar flow [2]. This abstract proposes a nonparametric method for estimating WSS from PC-MR images for more general application os WSS estimation to more complex vessel geometries and flow regimes.

## MATERIALS AND METHODS

Method: The WSS estimation method consists of: (1) determination of the vessel wall position, (2) fitting of a nonparametric function to the blood velocity measurements within the vessel, (3) approximation of the derivative of the blood velocity function at the boundary. Specifically, in step 1, the vessel interior wall pixels are automatically extracted from a magnitude image with an edge detection algorithm [3]. A closed, periodic smoothing spline curve is then fit to the boundary pixels to estimate the vessel wall at sub-pixel resolution and to compute the normal directions. Pixels from the PC-MR image that are within the boundary are segmented for fitting. In step 2, interior points are denoted as $(x_i, y_i, z_i)$ for $i$=1,…,$n$, where $x$ and $y$ indicate pixel location and $z$ indicates the corresponding velocity. Blood velocity is modeled as $z_i = f(x_i, y_i) + \varepsilon_i$ , where $\varepsilon_i \sim_{iid} N(0, \sigma^2)$, where we assume only that $f$ is a smooth function. An estimator for $f$ is found in a reproducing kernel Hilbert space denoted $\mathcal{H}_R$ with reproducing kernel $R$ such that

$$\hat{f}(x,y) := \arg\min_{f\in\mathcal{H}_R}\left(\sum_{i=1}^{n}(z_i - f(x_i,y_i))^2 + \lambda||f||^2_{\mathcal{H}_R}\right).$$

The unique solution follows from a more general result in [4] and involves a linear combination of the reproducing kernel evaluated at a point $t$ and the data, namely

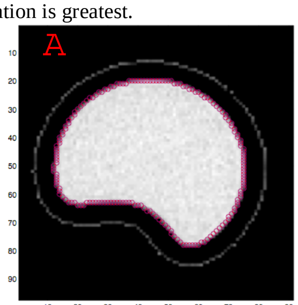
$$\hat{f}(t := (x,y)) = \sum_{i=1}^{n}\mathbf{c}_i R(t,t_i), \text{ where } \mathbf{c} := (\mathbf{\Sigma}+\lambda\mathrm{I})^{-1}(z_1,\cdots,z_n)', \ \mathbf{\Sigma} := [R((x_i,y_i),(x_j,y_j))]_{ij}, \ i,j = 1,\cdots,n.$$

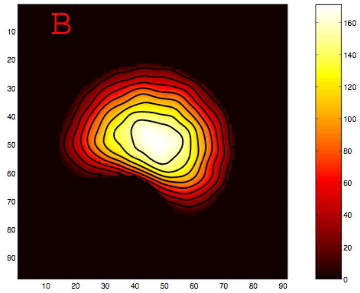
The function $R$ provides a model for spatial covariance and matrix $\mathbf{\Sigma}$ can be regarded as a spatial covariance matrix. In fact, the $R$, which is a symmetric positive definite function, defines a unique $\mathcal{H}_R$. Here, $R$ is selected from the Matern family of radial basis functions [5]. Let $\tau := ||(x_i,y_i)-(x_j,y_j)||$ be the Euclidean distance between two points. Then $R_\nu(\tau) := \exp(-\tau)\pi_\nu(\tau), \ \nu = 0,1,2,\dots$ is the Matern reproducing kernel for order $\nu$, where $\pi_\nu$ is a polynomial of a particular form [5]. The regularization parameter $\lambda$ and order $\nu$ can be objectively determined with generalized cross-validation [6]. In step 3, the estimated velocity function is used to evaluate the derivative at the boundary along the normal directions, which yields the wall shear rate (WSR). WSS is the product of the blood viscosity and WSR.

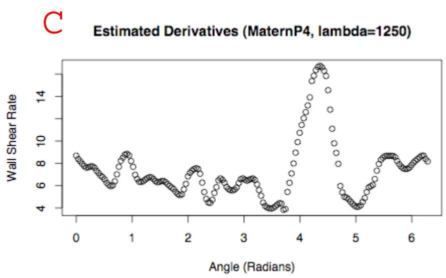
Experiment: A glass tube phanton was constructed with an indentation to provide a nonconvex cross-section. The phantom experiment was performed on 1.5 T Signa LX (GE Medical systems, Milwaukee, WI) scanner using a 2D phase contrast (PC) sequence. The glass tube was connected to a Cole-Parmer pump placed outside the scanner. The flow rate was set to 6.25ml/s. The parameters used for the scan were FOV=80 mm x 80 mm, matrix size= 512 x 512, NEX= 10, TR/Flip = 29ms/15° and slice thickness= 0.7mm.

## RESULTS

The proposed method was successfully applied to the phantom data even in the concave region of the phantom where parametric methods that depend on convexity or symmetry assumptions would typically fail. The boundary pixels determined by the Canny method are marked on the magnitude image in (A). The estimated velocity function along with contours is in (B). A regularization parameter of $\lambda$=1250 and Matern order $\nu$ = 4 were used to estimate the velocity function. Notice that the contours are more closely spaced near the indentation than in other regions of the tube. This indicates a greater rate of velocity change and, hence, greater wall shear stress. The estimated derivatives are plotted versus angle in (C). The angle is with respect to the center of the tube and increases counter clock-wise. There is a systematic trend in the estimates that reflects the shape of the tube. Note the large WSS between angles 4 and 5 radians. This is the region of the tube where the indentation is greatest.




## DISCUSSION

The main result of this abstract is to describe a nonparametric method and establish its feasibility for estimating WSS in blood vessels. Nonparametric function estimation in $\mathcal{H}_R$ does not require restrictive assumptions about the form of the blood velocity profile or symmetry of the vessel. In the phantom data, this method produced a good fit and sensible estimates of WSR along the entire vessel wall. While the Canny method was able to automatically identify boundary pixels from the phantom data, determining vessel boundaries *in vivo* might be more difficult. Others have shown the advantages of double inversion black blood MRI for identifying vessel boundaries [7]. One limitation of this study is the lack of a "gold standard" to confirm the WSR estimates. Measuring WSR with additional methods would provide confirmational evidence. While not considered in this abstract, the bootstrap is a procedure that can be used to estimate confidence intervals or standard errors for the WSS estimates. This method can also be extended to fit 3D images or a series of contiguous 2D images acquired along the flow direction. Additional phantom studies and *in vivo* studies are necessary to provide better understand the strengths and limitations of this nonparametric method and how they compare to other methods. The preliminary results from this study suggest the promise of the nonparametric method as a potential diagnostic tool.

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