Debye Potential based Method for the Analysis of Electromagnetic Fields inside a Lossy, Multilayered Spherical Head Phantom Excited by MRI Coils

F. Liu¹, S. Crozier¹, J. Li¹

¹School of Information Technology and Electrical Engineering, Brisbane, Queensland, Australia

Synopsis

Suitable phantom-based modelling and analysis is essential to MRI coil design and evaluation. This paper presents an analysis model of MRI coils in the presence of a multi-layered lossy sphere, roughly resembling the human head. The field calculation is based on a Debye Potential (DP) formulation and deals with a symmetric configuration in which the source is a circular loop carrying a harmonic-formed current at any frequency. Test examples have shown the capability of the proposed model.

Introduction

A variety of head models/phantoms have been developed. Usually, the head is approximated by a homogenous sphere [1-6]. These are invaluable for evaluating magnetic stimulation problems and RF probe analyses. However, as the human head is more like a stratified spherical structure in electrical properties, a multilayered spherical model might be a more effective model to rapidly evaluate the loading effects of biological structures in MRI. We therefore introduce here a generic spherical model which can be divided to be an arbitrary number of concentric spherical layers of constant dielectric properties. The proposed method works accurately over all the operating frequencies in MRI. After the electromagnetic fields (EMFs) are calculated, various imaging or antenna parameters can be investigated in a straightforward manner.

Debye Potential-based Solution for Ideal Current Loop

The coil/sphere structure is illustrated in Fig.1, where a current loop carrying uniform current is placed adjacent to a *L*-layered, lossy dielectric sphere. In this configuration, the Maxwell's equations can be expressed by $(\nabla^2 + k^2)\pi_m = 0$ [5], where π_m is the Debye potential (DP). The DP solution has the general form

 $\pi_{m} = \sum_{n=0}^{\infty} \left\{ j_{n}(kr), h_{n}^{(1)}(kr) \right\} P_{n}(\cos\theta) \text{ Where } j_{n}, h_{n}^{(1)} \text{ is the spherical Bessel and Hankel functions, respectively; } P_{n} \text{ is the associated Legendre polynomial. Without consideration of the zero-components: } E_{r}.E_{\theta}.H_{\phi}, \text{ the left EMFs can be expressed as}$

$$E_{\phi}^{(l)} = -\sum_{n=0}^{\infty} [C]_{n}^{(l)} [B]_{n}^{(l)} [DP]_{n}, H_{\theta}^{(l)} = \frac{1}{iw\mu_{0}r} \sum_{n=0}^{\infty} [C]_{n}^{(l)} [DB]_{n}^{(l)} [DP]_{n}, H_{r}^{(l)} = \frac{1}{iw\mu_{0}r} \sum_{n=0}^{\infty} n(n+1)[C]_{n}^{(l)} [B]_{n}^{(l)} [P]_{n} [DP]_{n} [DP]_{n$$

Where *l* denotes the layer number, $[C]_n^{(l)}$ is the expansion coefficients matrix; $[B]_n^{(l)}$, $[DB]_n^{(l)}$ are the matrixes of spherical Bessel function and its derivates; $[DP]_n$ is the derivate of the associated Legendre polynomial $[P]_n$. To determine the unknown elements of $[C]_n$, the boundary conditions at each spherical layer surfaces may be considered. After some rearrangement, the following matrix equation is obtained: $[A]_{(2L+2)}^n [C]_{(2L+2)}^n = [B]_{(2L+2)}^n$, where [A] is expressed by the Bessel and Legendre polynomial, and $[B]=[0, \dots, f(J)]$, and f(J) is the current source component. All the expansion coefficients can be obtained directly by solving this linear matrix system and then one can calculate the EMFs and other quantities of interest inside the sphere.

Simulations

To show the capability of the proposed method, both low- and high- frequency cases were studied. The tissue properties of the human head used in this work were obtained from the US Air Force Research Laboratory (http://www.brooks.af.mil/AFRL/HED/hedr/). Typically, the head can be approximated by a number of layers (white matter (WM), gray matter (GM), Skull, Muscle, Fat, Skin) from inner brain to the outer boundaries. For gradient field analyses, induced eddy currents were calculated by DP, Quasistatic finite difference (QSFD) [7] and FDTD methods, respectively. From the comparison as shown in Fig.1, it can be seen that all the methods yielded similar results and reflected the effect of inclusion of the highly-contrasted conductivity profiles on the eddy current distribution. Current density maximum values could be located deep within a layer even though the E-field

maximums may be at the surface. This type of layered information is important for realistic estimates and cannot be obtained from homogeneous isotropic models. To test RF fields, a head phantom consisting of 4-concentric spheres representing brain, CSF, skull and scalp tissue. A predefined current distribution along the surface coil was assigned and 1.5,4,7,11T fields were investigated as shown in Fig.2. It was found that the H-fields were disturbed less by the phantom at low frequencies such as 64MHz; however, as the frequency increases, the field profile is more strongly influenced, especially at positions near large contrasts of electrical properties.

Conclusion

The proposed DP approach enables one to rapidly assess the topology of field behaviour inside the objects excited by both the gradients and RF resonators. It is anticipated that the solutions/results will benefit the MRI coil design/analysis. Such analyses are intended as a rapid prototyping facility before full FDTD calculations are undertaken. **References**

[1] Hoult & Phil, J Magn Reson Imag., 12: 46-67 (2000).

[2] Keltner, Carlson, Roos, Wong , Wong & Buninger, Magn Reson Med., 22: 467-480 (1991).

[3] Collins, et al., Magn Reson Med. 47: 1026-1028, (2002).

[4] Strilka et.al., Magn Reson Imag, 16(7): 787-798 (1998).

[5] Jin, Electromagnetic analysis and design in magnetic resonance imaging, CRC, (1999).

[6] Bencsik, Bowtell & Bowley, Phy Med Biol. 47(4): 557-576 (2002).[7] Liu, Zhao & Crozier, IEEE Trans Biomed Eng 50 (7): 804-815 (2003).



Fig.1. Comparisons of the DP, QSFD and FDTD solutions for the induced eddy currents by a current loop in a seven-layered sphere along the radial distance at x,z=0.The loop (radius: 26cm) carried a uniform unity current and the frequency is 1kHz, the distance between the loop/sphere centre is 10cm. The radii of the spherical layers are 5,8,8.4,8.8,9.2,9.8 and 10 cm, respectively.

Fig.2. Comparison of the DP solution for the RF fields at different resonance frequencies (64,170,300,470MHz) in a 4-layered sphere (6,8,8.4,10cm) in the x=0 m plane.