Radiation Damping and Reciprocity in Magnetic Resonance Reception

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The signal to noise ratio (SNR) per voxel in MR imaging is (*1*) SNR = $\sqrt{(E_i/kT)}$, where E_i is the signal energy in the *i*th voxel and kT (for ideal reception) is the noise energy per point. Noise power is fixed for a given receive bandwidth, so relative SNR depends upon delivery of signal energy, dE/dt. This is given in the theory of (so called) radiation damping, in the classic (but problematic) paper of Bloembergen-Pound (*2*), who parameterize in terms of the coil Q and filling factor η – of which the latter is not readily measured in a practical NMR experiment. The Bloembergen-Pound definition of η (transformed to SI) units is the ratio of integrals: $\int \mathbf{B}_1 \cdot \mathbf{M} dV / \mathbf{M}_0 \cdot \int \mathbf{B}_1 dV$, rather than the conventional ratio of sample volume to coil volume (*3*). Alternatively, the reciprocity formula of Hoult-Richards (*4*) employs instead of these a figure of merit we call the *transducer efficiency* – $B_1(1)/\sqrt{R}$, with $B_1(1)$ the RF magnetic field at unit current, and R the probe resistance – or its equivalent and easily measureable form $B_1(P)/\sqrt{P}$ where P is power absorbed by the probe. The transducer efficiency varies as RF homogeneity, and is therefore a local figure of merit. Also, since (as is common in reciprocity arguments) the current is divided out (*5*) so there is no explicit oscillatory time dependence. We give below formulae for radiation damping in terms of the efficiency, and briefly consider inconsistencies in the original theory.

The Zeeman energy E_z following a pulse with tip angle θ is $-M_0 V B_0 \cos\theta$; its time derivative is the power delivered to the receiver, which is just the power available from the sample, assuming all receiver components are impedance matched:

$$\frac{dE_z}{dt} = M_0 V B_0 \sin\vartheta \frac{d\vartheta}{dt} = \frac{V^2}{4R} = \frac{\{\omega M_0 \sin\vartheta V B_1(1)\}^2}{4R}$$

where V is the emf of the precessing nuclei (*4*), R is the coil resistance, and we assume for simplicity a uniform B_1, so that the transverse magnetization is just $M_0 \sin\theta$. Alternatively, from the Bloch equations for the damping field due to receiver current in the coil:

$$\frac{dM_z}{dt} \equiv -M_0 \sin\vartheta \frac{d\vartheta}{dt} = -\gamma M_x B_1 = -\gamma \frac{\omega V\{M_0 \sin\vartheta B_1(1)\}^2}{4R}$$

where γ is the gyromagnetic ratio, and the result is essentially that of Eq. [1]. The factor of 4 in the denominator derives from the oscillator current of $V/2R$, and the assumption of a linear coil in the quasi-static regime, where rotating field strength is one half the linear. Also recall that $B_1(1)$ has units of tesla/amp. These equations are straightforwardly adaptable for inhomogeneous RF, by introducing a spatial integral (*3*) of the field over the sample volume. The predicted radiation damping constant is:

$$\frac{1}{\tau_d} = \gamma \frac{\omega M_0 V B_1(1)^2}{4R}$$

for which we obtain a value τ_d of 2.4 sec, for a litre of water in a shielded bird cage or TEM head resonator at 3.0 T (i.e. $B_1(1) = 4$ μT and $R \sim 20$ ohms.) Our formula differs in appearance from the Bloembergen-Pound result $1/\tau_d = 2\pi\gamma\eta Q M_0$, for which they obtain the rather unrealistic value of $\tau_d = 0.03$, for water protons at a field of 0.7 T, with Q of 100 and $\eta = 1$. Gueron (*6*) has re-written their formula in SI units as $1/\tau_d = \mu\gamma\eta Q M_0/2\pi$, and notes that the quantity ηQ characterizes the sensitivity of the probe; it is roughly analogous to the efficiency (*vide supra*). However, using the definition of inductance $L = (1/\mu)\int \mathbf{B}_1(1) \cdot \mathbf{B}_1(1) dV$, with $Q = \omega L/R$, shows that this corresponds only approximately to the reciprocity result, since the integrals for L are over all space, and those in reciprocity formulae only over the sample. The filling factor is supposed to compensate for this; but it is not all apparent that it does. It is our belief that the radiation damping constant should be habitually written in a form which derives directly from reciprocity , possibly that of Eq. [3], provided its validity can be generally established.

References:

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