Theory of the Quality Factor for a Lossy Dielectric Cylinder in a Cylindrical Resonator

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The role of dielectric effects in high field MRI is unquestioned; but the relative importance of dielectric resonance is still debated. (*1-3*), The significance of the quality , or Q factor of a lossy dielectric has been emphasized (*4*) but Q itself has not been calculated in detail. We therefore present a model of a lossy dielectric cylinder inside a long, cylindrical RF antenna, of bird cage or TEM type, allowing exact calculation of the dielectric Q. Our model is two-dimensional, and similar to that of Spence and Wright (*5*), except we assume a field of TEM character inside the empty antenna.

We first solve for the vector potential of a single current carrying element; the result for the full antenna is obtained by azimuthal shifting, weighting, and adding.. The excitation is given by expansion of the line-current potential (6); the shield is modelled by an image current. The induced and scattered potentials at azimuth zero are:

$$\boldsymbol{A}^{cyl} = \frac{\boldsymbol{e}_z \mu_0}{2\pi}\left[\sum_{n=1}^{\infty} B_n J_n(kr)\cos n\vartheta + B_0 J_0(kr)\right] \qquad r < a \qquad [1]$$

$$\boldsymbol{A}^{scat} = \frac{\boldsymbol{e}_z \mu_0}{2\pi}\left[\sum_{n=1}^{\infty} A_n \left\{\left(\frac{r_s}{r}\right)^n - \left(\frac{r}{r_s}\right)^n\right\}\cos n\vartheta + A_0 \ln r\right] \qquad a < r < r_s \qquad [2]$$

where r_s is the radius of the resonator shield, A_n and B_n are bulky coefficients, and other symbols have the usual meanings. Collin defines (*7*) the Q of a dielectric:resonator:

$$Q = \frac{\omega_0 (W_1 + W_2)}{(P_1 + P_2)} \qquad [3]$$

where W_1 and W_2 are stored energies inside and outside, P_1 is the internal dissipated power, and P_2 the radiated power. All of these are calculable in our model, due to confinement of the fields by the shield. Also, for a resonator of N elements, the dissipation in the shield is

$$P(\omega) = \frac{\pi s(\omega) N}{2 r_s} \sum_n n^2 \, | A_n |^2 \qquad [4]$$

where $s(\omega)$ is the frequency dependent surface resistance, and other symbols have been defined. By itself, this yields a $Q \sim 1000$; we therefore neglect the radiative loss. Then calculating stored energies and losses from integrals of the squared fields, and using the equality of stored electric and magnetic energies, we arrive at the simple result $Q = \omega_0 \varepsilon \varepsilon_0 / \sigma$, where ω_0 is the frequency, ε_0 and ε are free space permeability and relative dielectric constant, and σ is the conductivity.

Figure 1 shows rotating flux plots for the loaded resonator, which illustrate the transition from an underdamped (Q>0.5) to overdamped (Q < 0.5) condition. Note the concentration of flux at the center in the former, and its expulsion (skin effect) in the latter (*4*). Figure 2 illustrates the resonant shift of the dielectric inside the cylinder; refer to the legend for details.

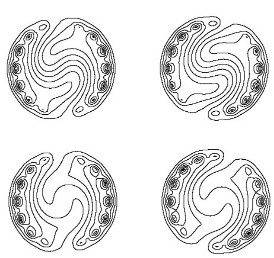

Figure 1: Rotating magnetic flux plots (time exposures) at 128 MHz for a shielded resonator of 16 elements. with dielectric load. Above $Q = 1.0$ ($\varepsilon = 80$ $\sigma = 0.7$); below $Q = 0.35$ ($\varepsilon = 80$ $\sigma = 2$); shield radius 17.7 cm, elements on 14.6 cm bolt circle, dielectric cylinder $r =$ 9.25 cm.

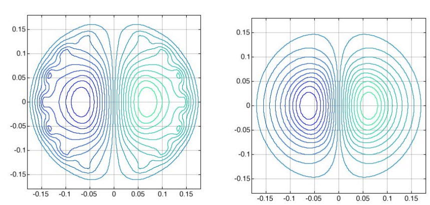

Figure 2: Flux plot of real part of vector potential for 16 element resonator with lossless dielectric load , $\varepsilon = 80$; dimensions as in Fig. 1. Left: 142 MHz (free space resonance of cylinder; contour interval is full scale/20); current elements and shield are well visualized. Right: 157 MHz, true (shifted) resonant frequency of dielectric inside shield; contour as at right; resonator elements no longer visualized, indicating greater concentration of flux inside dielectric at true resoanance, and the shift in load reactance due to shield

References:

1 P. Roschmann,**,** *Med. Phys.* **14**, (1987), 922.
2. D. I. Hoult, *J. Magn Reson Imag*. (2000)**12**, 46.
3. J. T. Vaughan *et. al. Magn. Reson. Med*. (2001)**46**, 24.
4. J. Tropp, “Proceedings of the ISMRM” (2002) 912.
5. D. Spence & S. Wright, *Magn. Reson. Eng*. (2003)**18B** 15.
6. W. Smythe, “Static and Dynamic Electricity” 3rd Edn. McGraw Hill (1968) Ch. 4.
7. R. Collin “Field Theory of Guided Waves” 2nd Edn. IEEE (1991) 459 ff.