An Analytical Model of Gradient Coil Heating

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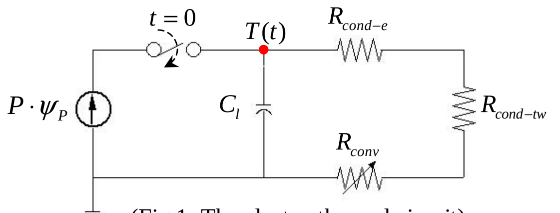
Synopsis Gradient coil heating has become a major barrier to achieving higher spatial and temporal resolution in MR imaging. It would be ideal to predict the thermal properties of gradient coils before they are built so that improvements can be made at the design stage. In this study, we developed a method to model the temperature response of multi-layer, water-cooled gradient coils. The model is fully analytical and derived just from the known material properties and coil dimensions. Actual heating experiments were carried out to evaluate the accuracy of the model. Results prove that this model is a practical tool for gradient coil design.

Methods The temperature response (*T(t)*) of a gradient coil to a power input (*P*) can be effectively modeled as the step function response of an electronic RC network, which consists of a limited number of components (Fig 1) [1]. These electro-thermal components are: the total thermal capacitance (C_l) of the coil; the conductive thermal resistance (R_{cond-e}) of the epoxy; the conductive thermal resistance of ($R_{cond-tw}$) cooling tube wall; the convective thermal resistance (R_{conv}) of the water flow; the power scaling factor (ψ_P) addressing the non-uniformity of power distribution.

The values of C_l, R_{conv} and $R_{cond-tw}$ are functions only of material properties and flow condition, and therefore can be treated constant throughout the coil. However, the deposited power (*P*) is strongly dependent on the local wire density. Therefore, we introduced the power scaling function, ψ_P, to account for the variation of wire density. R_{cond-e} is determined by the 2D heat transfer between the wire pattern and cooling tubes, which are both embedded inside the epoxy. For simplicity, we consider only the "worst case" scenario – temperature at the coil's "hotspot", where the wire density is the highest. ψ_P is estimated as the ratio of the wire density at the hotspot to the average density of the wire pattern. R_{cond-e} is estimated using 2D graphical analysis of long rods buried in a semi-infinite medium at a certain depth [2].

A three-layer (X, Y and Z axis) gradient coil was constructed in our lab. The three wire layers were interleaved with four copper tube cages (cooling layers) and potted in thermally conductive epoxy (DURAPOT 865). Values of the thermal capacitance and resistances were computed for our gradient coil using the analytical formulas developed, and are shown in Table 1. Note that R_{conv} is the highest of the three resistances, and that $R_{cond-tw}$ is negligible compared to the other two.



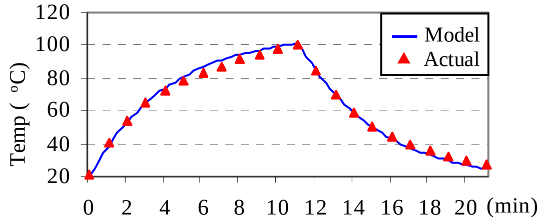
(Fig 1: The electro-thermal circuit)

Table 1: Estimated values of the electro-thermal components

Axis	R_{cond-e}	$R_{cond-tw}$	R_{conv}	C_l	ψ_P
	$\times10^{-3} K/W$			$W\cdot s/K$	
X	0.94	0.001	2.87	50349.6	2.47
Y	1.18				2.43
Z	0.90				2.36

Results A thermocouple was attached to the hotspot of each wire layer. MR microscopy imaging was carried out and the temperature at the most gradient-intensive axis (readout) was recorded. In the case of using Z-axis as readout, the RMS current to the Z-axis was 70A. The inlet temperature of the water was 11.7 °C. Fig 2 shows the comparison of the estimated and measured temperature response during the scan.

The experiment was repeated three times using a different axis for readout gradient in each scan. The accuracy of the model was depicted as the RMS error between the estimated and actual temperature over the entire measuring period (heating and cooling). As shown in Table 2, the RMS errors of these three experiments were less than 4°C, and the deviation between the estimated and actual values of maximum temperature were less than ± 5°C.



(Fig 2: Heating at the Z-axis hotspot)

Table 2: Model evaluation

Read Axis	Maximum temperature (°C)		RMS error (°C)
	Estimated	Actual	
X	96	91	4.0
Y	80	79	1.5
Z	101	101	2.1

Conclusion Results shown above demonstrate that a simplified mathematical model based on a small number of physical parameters can effectively simulate the temperature response of a gradient coil under the influence of current pulses heating and water-cooling. The model also identified the most important factors having direct bearing on the coil temperature: specifically, that the convective heat transfer from cooling tube wall to cooling fluid is the bottleneck in heat transfer for the gradient coil that we studied. Such an analytical model yields immediate and practical guidelines for improving the thermal design of high-strength gradient coils. Finally, the model makes it possible to predict coil temperature under widely different operational conditions, thus ensuring hardware safety and providing a useful tool for guiding scan prescription.

Acknowledgements NSERC, CIHR and GE Medical Systems

References [1] Karlekar et al., 1977 [2] Incropera et al., 1996