Stream Function Method for Design of Arbitrary-Geometry Gradient Coils

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Introduction

Over the past several years a variety of theoretical design methods for the construction of gradient coils have been developed. In his paper, Turner [1] considers a cylindrical coil structure and represents the inductance in terms of a current distribution over the surface. He then minimizes the inductance subject to the magnetic flux density meeting a desired field distribution in the region of interest (ROI). In another paper [2] D. Green *et al.* minimize a weighted combination of power, inductance, and the L2 norm between actual and desired field. Representing the current as a Fourier series they find optimal coefficients that minimize the associate cost function.

However, despite their apparent successes, all these methods suffer from a common disadvantage: they are only applicable to particular coil geometries such as cylinders, planes, or single and bi-planar surfaces. In our paper, we describe a new approach for the coil design that is largely independent of the shape of the current-carrying surface. We will demonstrate the success of our approach by designing a crescent G_x gradient coil.

Theory

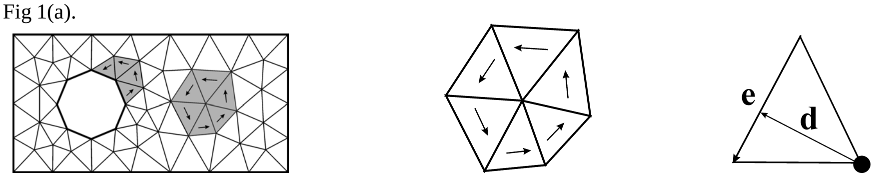
For simplicity, we consider only gradient coil consisting of just one surface. Next, we introduce a cost function Φ as follows:

$$\Phi = \frac{1}{2}\sum_{k=1}^{K} W(\mathbf{r}_k)\left(B_z(\mathbf{r}_k) - B_{des,z}(\mathbf{r}_k) + B_{off,z}\right)^2 + \alpha W_{magn} - \lambda_x M_x - \lambda_y M_y - \lambda_z M_z \qquad (1)$$

where $W(\mathbf{r})$ is a weight function, $B_{des,z}(\mathbf{r})$ is the *z*-component of the desired magnetic field, W_{magn} is magnetic energy of the current, α is a weight coefficient, and M_x, M_y, M_z are the components of the torque $\mathbf{M}$, which is calculated with respect to the origin as a fixed point. The expressions for W_{magn} and $\mathbf{M}$ can be cast in the form

$$W_{magn} = \frac{\mu}{8\pi}\int_S\int_{S'} \mathbf{J}(\mathbf{r})\cdot\mathbf{J}(\mathbf{r}')\frac{1}{|\mathbf{r}-\mathbf{r}'|}dSdS', \quad \text{and} \quad \mathbf{M} = \int_S \mathbf{r}\times\left(\mathbf{J}(\mathbf{r})\times\mathbf{B}_0(\mathbf{r})\right)dS \qquad (2)$$

Let us consider a surface shown on Fig 1(a).



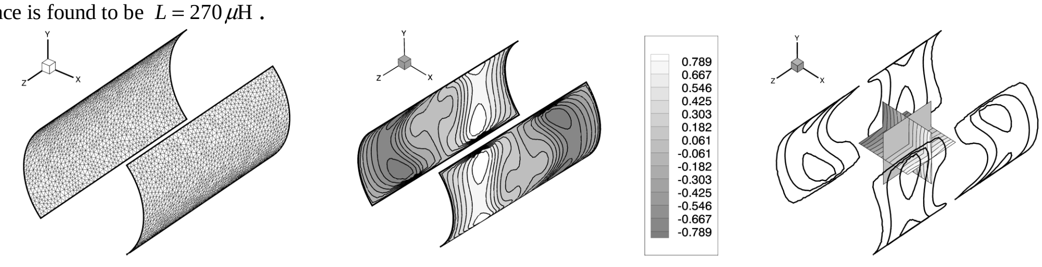
(a) (b) (c)

Figure 1: Current element definition.

We associate a current with each node as shown in Figs. 1(b) and (c). For each triangle belonging to a chosen node we define a basis function as $\mathbf{f}(\mathbf{r}) = \mathbf{e}/(|\mathbf{e}||\mathbf{d}|)$. All basis functions are assumed to rotate in the same (clockwise or counterclockwise) direction. The surface current then can be approximated as $\mathbf{J}(\mathbf{r}) \approx \sum_{n=1}^{N} I_n \mathbf{f}_n(\mathbf{r})$. Furthermore, we express $B_z(\mathbf{r})$ and W_{magn} in terms of currents I_n and obtain a system of linear equation for I_n, the offset field $B_{off,z}(\mathbf{r})$, and, if necessary, for λ_x, λ_y, λ_z. It is worth noting that not all I_n are independent: nodes belonging to the same boundary share the same current value.

Example of coil design

To demonstrate how the algorithm works, we consider a coil composed of two plates, each of which has a size of 20×10 cm. These plates are curved with a radius of *R*=6.5 cm and positioned side-by-side. The coil is next discretized into triangular mesh as seen in Figure 2(a). In total, the coil is composed of 2833 nodes and 5414 triangular patches. The resulting stream function distribution is shown in Fig. 2(b). One of the possible wire patterns is shown in Fig. 2(c). If we require $R = 1\Omega$ as coil resistance then each groove in Fig. 2(c) contains 9 turns of AWG-20 wire. The gradient strength in the center of ROI is 28.5 G/cm for a drive current of 100A. The total coil inductance is found to be $L = 270\,\mu\text{H}$.



(a) (b) (c)

Figure 2: G_x crescent coil design steps. (a) mesh, (b) stream function plot, (c) actual wire pattern and magnetic field.

Conclusion

This paper presents a new stream function method for the design of single- and multi-surface gradient coils. The method is formulated in such a form as to make it applicable to a wide variety of shapes and geometries. Once the triangular mesh of the desired coil structure is established, it can be fed into our computational algorithm to determine all rotational current elements. As a result, almost any arbitrary single or multiple surface geometry can be used as a basis of computing an optimal wire configuration based on performing a constraint minimization of a special cost function.

[1] R. Turner, "Minimum inductance coils," Journal of Physics E: Sci. Instrum. 21, pp. 948-952, 1988.
[2] D. Green, R.W. Bowtell, P.G. Morris, "Uniplanar Gradient Coil for Brain Imaging," Proc. Intl. Soc. Mag. Reson. Med. 10, 2002.