Exact amplitude calibration of hyperbolic secant RF pulses

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Introduction

Calibration of the B_1 amplitude of adiabatic RF pulses is usually either omitted, the use of a very high transmitter power ensuring adiabaticity, or based on empirical equations [2, 4] or manual calibration. These approaches are of limited utility if the inversion efficiency needs to be known precisely; the minimal acceptable B_1 amplitude is sought; or less than complete inversion is required. We present here both exact and approximate equations for the B_1 amplitude of a hyperbolic secant (HS) RF pulse required for a given population inversion and bandwidth. These equations are derived from an analytical solution of the Bloch equations [1] and are valid for arbitrary flip angles.

Methods

The Bloch equations have been solved analytically for a variety of RF pulses, including pulses driven by the hyperbolic secant / hyperbolic tangent amplitude and frequency modulation functions [3, 1]. Specifically, isochromats that are fully relaxed ($M_z = M_o$) at $t = -\infty$ and that experience an on-resonance amplitude and frequency modulated RF pulse of the form

$$AM(t) = \frac{1}{2\pi\gamma}\frac{a}{\pi}\frac{2\beta}{T_p}\,\mathrm{sech}\left(\frac{2\beta}{T_p}t\right) \tag{1}$$

$$FM(t) = \frac{b}{\pi}\frac{2\beta}{T_p}\tanh\left(\frac{2\beta}{T_p}t\right) \tag{2}$$

undergo a population inversion of [1, 2]:

$$P_e = \mathrm{sech}^2\left(\frac{\mathrm{b}}{2}\right)\left[\sinh^2\left(\frac{b}{2}\right) - \sinh^2\left(\frac{1}{2}\sqrt{b^2-a^2}\right)\right], \tag{3}$$

where $P_e = (M_0 - M_z)/(2M_0) = \frac{1}{2}(1-cos(\alpha))$, α being the equivalent flip angle. The amplitude, bandwidth and cutoff parameters of the pulse (a, b and β) have units of radians with the pulse duration T_p and time $t \in [-T_p/2; T_p/2]$ in seconds. The pulse modulation functions, AM and FM, are expressed in Tesla and $\frac{\mathrm{rad}}{\mathrm{s}}$, respectively, and γ is the gyromagnetic ratio in Hz/T. The bandwidth parameter b used here is related via $b = \pi\mu$ to the dimensionless bandwidth parameter μ of the notation used in [3]. Equation 3 can be inverted to obtain an analytical expression for the peak RF amplitude (in Tesla) that is necessary to achieve a desired degree of population inversion P_e on-resonance:

$$AM(0) = \frac{\beta}{\pi^2\gamma T_p}\sqrt{b^2 - 4\mathrm{acosh}^2\left(\cosh\left(\frac{\mathrm{b}}{2}\right)\sqrt{1-\mathrm{P_e}}\right)}. \tag{4}$$

In practice, it is often desirable to calculate the achievable bandwidth, given a required population inversion and a maximum RF amplitude. While Equation 4 can not be solved for b analytically, it may be very well approximated for moderate to large values of b:

$$AM(0) \approx \frac{2\beta}{\pi^2\gamma T_p}\sqrt{L}\sqrt{b-L}, \tag{5}$$

where $L = \ln\left((1-P_e)^{-\frac{1}{2}}\right)$, yielding an equation that can be readily solved for the bandwidth parameter.

Results

Figure 1 shows the RF amplitude parameter a as a function of the square root of the bandwidth parameter b, for the exact solution (thick black line, Equation 4) as well as the approximation (thin black line, Equation 5) for two target population inversions corresponding to flip angles of 90º and 175º. The flip angle optained with the approximation agree to within 0.1 % with desired flip angles for all bandwidth parameters and population inversions of practical importance ($b > 8.25$, $\mu > 2.63$, $P_e < 0.998$). Figure 1 also shows previous empirical results: extensive numerical simulations of the Bloch equations led to the relations plotted in dashed red lines [4, Eqs. 8-10], providing acceptable results for inversion, but considerably overestimating amplitude for saturation (equivalent flip angle of >110º). The power law derived in [2] (dotted blue line) is based on an empirical solution of Equation 3 for the amplitude parameter necessary to reach an “adiabaticity threshold”.

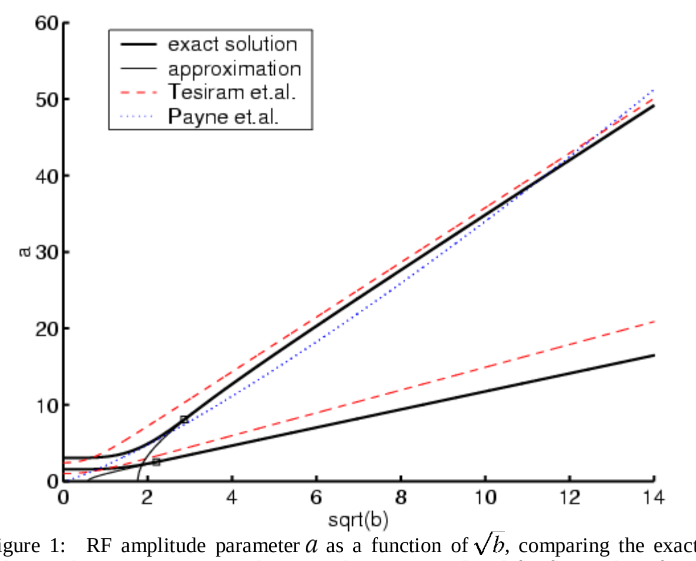

Figure 1: RF amplitude parameter a as a function of $\sqrt{b}$, comparing the exact solution, the approximation and empirical expressions [2, 4] for flip angles of 90º and 175º. At the black squares, the approximated RF amplitude deviates by 1% from the exact solution.

Conclusion

Empirical calibration equations fail to provide the necessary accuracy for flip angles below near-complete inversion, whereas the equations presented here provide the exact amplitudes required for HS pulses of arbitrary flip angles. The pulses can thus be used to obtain highly selective saturation, despite not being adiabatic at these amplitudes and thus requiring precise calibration like conventional amplitude-modulated pulses.

References

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