Analytical Approach for Estimating Mutual Inductance of Arbitrary Shaped RF Coils

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Introduction: RF coils are a critical component of magnetic resonance imaging that must be designed with constraints of form and SNR in mind. In these cases, estimating the inductance is needed in order to achieve appropriate tuning and matching for optimal performance. Similarly, when phased array coils are beneficial, it is further necessary to reduce coupling among these coils thereby requiring an estimate of their mutual inductance. One such application is the design of breast coils. We have been exploring the use of supine breast MRI in which the breast geometry can be highly variable. Our interest is to create a coil that covers the breast in a close fitting form which providing optimal coverage of the breast with phased array geometry.

Various decoupling methods have been proposed based on either a partial overlap over the coil members or through external decoupling circuitry. The mutual inductance between any pair of coils depends on their shape, size and their relative position. Calculating this parameter is routinely performed by a numerical integration corresponding to the coil geometry. In this paper, we introduce a different formulation of the Neumann integral that is efficient and accurate.

Method: The mutual inductance between two conductors can be written as: $M = \frac{\mu_0}{4\pi I_1 I_2}\int_{v_1}\int_{v_2}\frac{J_1(\vec{r}_1)\bullet J_2(\vec{r}_2)}{|R_{12}|}dv_1 dv_2$ where J is the current density and $|R_{12}|$ defines the spacing between current filaments. For RF coils, the cross section of wire or strip can be considered as constant along the entire current path so that M can be written in the form: $M = \frac{\mu_0}{4\pi}\int_{l_1}\int_{l_2}\frac{d\vec{l}_1 \bullet d\vec{l}_2}{|R_{12}|}$. This integral is referred as Neumann integer and can become complex for irregular shapes. However, this integer became relatively simple for two straight wires, as the dot product becomes the simple multiplication: $M_{12} = \frac{\mu_0 \cos(\theta)}{4\pi}\int_{l_1}\int_{l_2}\frac{d\vec{l}_1 d\vec{l}_2}{|R_{12}|}$. Campbell [1] provided an analytic solution for this integer. Building on this representation of the Neumann integral, we represent a curvilinear coil element as a series of straight elements of varying lengths. This then leads to an expression for the mutual inductance between any two irregular shaped coils which can be estimated by: $M = \sum_{i=ABCDE...}\sum_{j=abcde...} M_{ij}$ where M_{ij} denotes the analytical formula of Neumann integral between section *i* and *j* and *ABCD… abcd…* represented sections of each coil. Similarly, the inductance of a single arbitrary coil structure can be estimated as:

To demonstrate the use and accuracy of this technique, we test this on a spherical coil as shown in Figure 1. In this case, we define the coil in a series of approximations to the different segmentations of a coil with the shape of part of hemi-sphere. In this calculation, we estimate the mutual inductance between two such coils placed at varying angular displacement around the sphere as shown in Figure 2. By increasing the number of straight elements for each curved portion of the coil, the estimate of mutual inductance converges.

Results: The mutual inductance was calculated between the two coils. Segmentation started from one segment per side to over five segments per side depending on requirement. For this example, it was found to converge with 4 to 5 straight elements per curved section. Generally speaking, 3 elements provided enough accuracy except for very small angles where coupling become very strong.

Discussion and Conclusion: This analytical approach provided an efficient way of estimating mutual inductance between arbitrary shaped coils. We note that the straight element approximation converges rapidly toward to accurate value and that only a few number of such straight segments are needed to achieve accurate measures of mutual inductance. With this approach, the design of patient specific coil structure will become more automated and practical in the clinical setting.

References: [1] G. Campbell, Phys. Rev. V5. P452, 1915

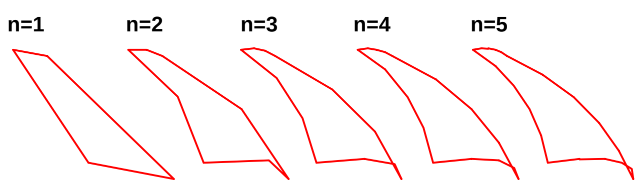

Figure 1. Different segmentations of a coil, which shape is part of a sphere with azimuth opening of 60°, elevation opening of 45° and four sides connected with great circle lines. Calculation showed that n>=3 provided enough accuracy.

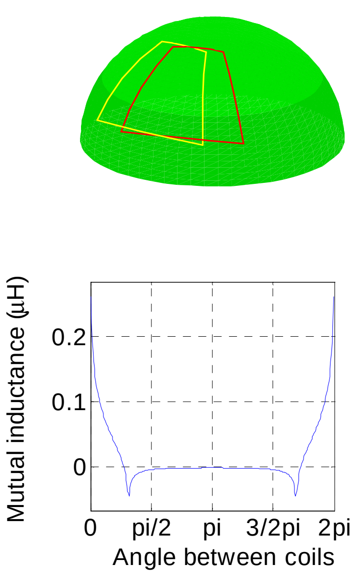

Figure 2 Top: two coils on a sphere with an angle of 18 degree. Bottom: Calculated mutual inductance between two coils as function of angle.

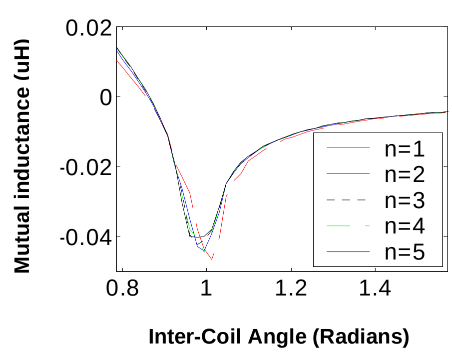

Figure 3 A detailed view of the mutual inductance for different segmentations with increasing number of straight elements.