A New Approach for an Analytical Formulation of the NMR Signal Using the Principle of Reciprocity

T. S. Ibrahim[1]

[1]School of Electrical and Computer Engineering and The OU BioEngineering Center, The University of Oklahoma, Norman, Oklahoma, United States

Introduction: The issue of the received signal in MRI [1,2] was brought to attention lately by several groups around the world [3,4]. In 2000, D. I. Hoult published an article that elegantly described the strength of the MRI signal using the principle of reciprocity [4]. In here, a theoretical derivation for the principle of reciprocity applied to an MRI experiment is presented. The approach shown in this paper is different than the one in [4]. In the current derivations, the physical structure of the receive and transmit coils will be general. In addition, excitation and reception will be assumed to be performed by two different coils. The mathematical derivation shows the validity of using the principle of reciprocity when it is applied with the proper conditions and it verifies the results provided in [4].

Methods and Derivations: The signal that contributes to an MRI image is related to the induced transverse magnetization which could be represented in a susceptibility tensor formulation [5] as follows:

$$\begin{pmatrix} 1-j\frac{\gamma\mu_0 M_0 T}{2} & -\frac{\gamma\mu_0 M_0 T}{2} & 0 \\ \frac{\gamma\mu_0 M_0 T}{2} & 1-j\frac{\gamma\mu_0 M_0 T}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

It is apparent that the matrix is not reciprocal and the standard reciprocity equation can not be applied if reciprocity is to be used in explaining the excitation of the spins. Rather, a more generalized reciprocity equation needs to be utilized in such a case.

The analysis shown in here will assume that fields could be represented on a macroscopic level. From Maxwell's equations in the time domain, the derivative of the magnetic flux density with respect to time is equivalent to a magnetic current source, more precisely, displacement magnetic current density. In the frequency domain, the magnetization is given as: $MC_{2x} = -iMC_{2y} = S$

In the MRI experiment, volume or surface coils are utilized for reception. The geometries of these coils, such as volume RF head resonators, could be of great complexity. Figure 1 displays the reciprocity problem for the MRI experiment, where a high pass birdcage resonator is used as the receiver coil. The signal measured with the resonator induces voltage between the coil terminals. These terminals, enclosed in surface Sexc (Figure 1) which is located in a transmission line used for excitation, (if the coil was used for that purpose) or reception. In here, a general case is considered: a coil is used to excite the spins and a different coil is used for the reception of the NMR signal. In the frequency domain, is it possible to apply the standard reciprocity equation? Interestingly, enough, the reception problem could be now considered as a reciprocal problem. We have time harmonic electromagnetic sources (transverse magnetizations due the spins), which are exciting RF probes. The entire process could be utilized within the context of electromagnetics also under the assumption that biological tissues are isotropic materials in the MR RF frequency range. As such, there is no condition for anisotropy let along non-reciprocal behavior. However, the standard reciprocity equation can not be directly applied for a different reason. For the forward problem, if the birdcage coil (Figure 1) is used for transmission, the magnetic current sources (transverse magnetization) are not existent (Figure 1). Clearly, $M_z=M_o$ before the RF coil is applied. A magnetic current source is equal to dM/dt. Therefore, fields 1 and 2 are produced by the birdcage coil in the absence of MC2. During reception, however, the fields 2 are produced by MC_2 while the birdcage coil is present. To resolve this issue, let surface Sv encapsulates volume V which boundaries are defined by surfaces S, Sexc, and Scoil. Surface Scoil tightly encapsulates the metal parts of the coil and the transmission line as shown in Figure 1. From divergence theorem,

$$\iiint_V \nabla.(\hat{E}_1\times\hat{H}_2-\hat{E}_2\times\hat{H}_1) = -\iint_{Sv}(\hat{E}_1\times\hat{H}_2-\hat{E}_2\times\hat{H}_1).\hat{n}ds$$

where Sv = Scoil+Sexc+ Σ . The integral vanishes at surface Σ due to Summerfield's radiation condition. If the coil is composed of perfect conductors, at surface *Scoil*, The tangential electric fields vanish. The reciprocity equation reduces to:

$$\int_v \hat{M}C_2.\hat{H}_1 dv = -\iint_{Sexc}(\hat{E}_1\times\hat{H}_2-\hat{E}_2\times\hat{H}_1).\hat{n}ds$$

The fields at surface *Sexc* are within the transmission line (middle of it). It is fair to assume a dominant TEM mode propagating within the line with dominant orthogonal vector mode functions (unit vectors) where: $\hat{e}\times\hat{h}=\hat{n}$ and ***n*** is prepé ndicular to the surface Sexc.

With MC2 being infinitesimal, the reciprocity equation is modified as follows:

$$\hat{M}C_2.\hat{H}_1 = -\iint_{Sexc}(V_{tr}\hat{e}\times I_{rc}\hat{h}-V_{rc}\hat{e}\times I_{tr}\hat{h}).\hat{n}ds$$

$$\hat{M}C_2.\hat{H}1 = V_{tr}I_{rc}-V_{rc}I_{tr}$$

where V_{tr} and I_{tr} are the voltage and current modal amplitudes in the transmission line when the coil is transmitting, and V_{rc} and I_{rc} are the same quantities when the coil is receiving. For the open circuit condition, $I_{rc} = 0$, and the open circuit voltage is obtained:

$$V_{oc} \approx -\frac{S\frac{(\hat{B}_1^+)}{|B_1^+|}.\hat{H}_1}{I_{tr}}$$ and it follows: $$V_{oc} \approx -\frac{SH_1^-}{I_{tr}}$$

In the preceding equation: the $B_1^+$is a fictitious component of the B_1 field, induced by the birdcage coil (Figure 1). This component would excite the magnetization of interest if the B_1 field was used for excitation. The definition of H_1^- field is provided as the circularly polarized component of the field where its sense of rotation is always opposite to that associated with the B_1^+ field. The current source MC_2 could have been excited by the birdcage coil, by any other coil, or by any combination of coils. Therefore, regardless of the spatial configurations used in this problem, the received signal in MRI is a function of the magnetization and the H_1^- field described above. This expression agrees with the formulations obtained in [4].

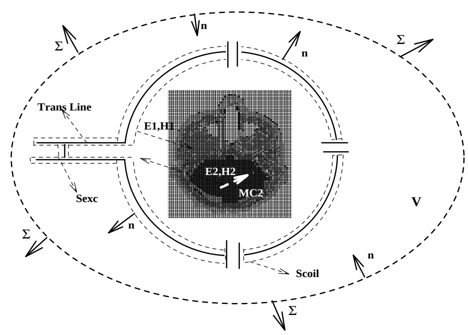

Figure 1. Reciprocity in MRI Experiment. A high pass birdcage coil is used as a receiver coil. The voltage induced by the magnetization is measured at *Sexc*. *Sexc* is included in a transmission line that supports a TEM mode.

References:

[1] C. N. Chen, V. J. Sank, S. M. Cohen, and D. I. Hoult, *Magn. Reson. Med.*, vol. 3, pp. 722:729, 1986.

[2] G. H. Glover, E. C. Hayes, et al, *J. Magn. Reson.*, vol. 64, pp. 255:270, 1985.

[3] C. M. Collins, Q. X. Yang, et al, *Magn. Reson. Med.*, vol. 47, pp. 1026:1028, 2002.

[4] D. I. Hoult, *Concepts in Magnetic Resonance*, vol. 12, pp. 173:187, 2000.

[5] A. Abragam, "The Principles of Nuclear Magnetism", Clarendon Press, Oxford, 1961.