A High-Frequency Analytical Model of the Shielded Birdcage Coil Using Multiconductor Transmission Line Theory

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Introduction

Analytical models of the birdcage coil found in the literature consist mostly of lumped-element circuits and therefore assume that the wavelength is at least 10 times larger than the dimensions of the coil. Birdcage coils have been constructed to operate in excess of 200 MHz[1], where this assumption is no longer valid even for head-sized coils. In such situations the lumped-element models can no longer be used with accuracy to determine the relationship between electrical parameters and resonant spectrum. Furthermore, high-frequency effects can alter the RF field distribution. It has been observed[2] that significant variations in B_1 field intensity along the axis of the coil are determined by its axial propagation constant, k_z, resulting in leg currents which vary along the length. For a high-pass coil, for example, a current maximum occurs at the central transverse plane, concurrently with an electric field minimum[3], while the currents taper off toward the ends of the coil as $\cos k_z{\cdot}z$ in perfect analogy with a resonant transmission line. This effect limits the maximum practical length of a birdcage coil at a given frequency to a small fraction of the axial wavelength, not only because the region of uniform B_1 is axially limited, but also because longer lengths bring the coil near the self-resonant limit[4] consequently becoming unstable and extremely difficult to tune.

The model presented below is able to account for the finite phase velocity of electromagnetic waves by considering the birdcage coil as being a section of a multiconductor transmission line (MTL) that is terminated at either end by capacitors joining adjacent transmission line elements. Such a model has been used with success to describe the TEM resonator[5,6] where each leg of the resonator is shunted capacitively to the shield instead of to its neighbors. It differs from the standard lumped-element models[7] not only by accounting for wavelength effects but also for *distributed* coupling among the longitudinal conductors which includes electric coupling in addition to its well-known magnetic counterpart. Furthermore it predicts the existence of modes which are not predicted by the lumped-element models but which are readily observed at frequencies greater than the frequency for which the length of the coil equals one-half wavelength.

Mathematical details

The condition for resonance of an N-element multiconductor transmission line terminated symmetrically is that the determinant $\left|\mathbf{I}_N - \mathbf{\Gamma}\cdot\mathbf{\Gamma}\exp(-2i\omega d/c)\right|$ approach zero. Here d is the length of the line, c is the speed of light in free space (for simplicity the coil is assumed to be unloaded), $\mathbf{I}_N$ is the identity matrix of order N and $\mathbf{\Gamma}$ is the reflection coefficient matrix, which is calculated as $(\mathbf{I}_N - \mathbf{Z}_0\mathbf{Y})(\mathbf{I}_N + \mathbf{Z}_0\mathbf{Y})^{-1}$, where $\mathbf{Z}_0$ is the characteristic impedance matrix and $\mathbf{Y}$ is the termination admittance matrix[5,6]. The latter are calculated, respectively, from the inductance-per-unit-length matrix of the line, and the end-ring capacitances and inductances. For a birdcage coil $\mathbf{Y}$ has elements on the main diagonal as well as on the two diagonals adjacent to it, while in the case of the TEM coil $\mathbf{Y}$ is strictly diagonal. More complex expressions can be obtained for low-pass coils or similar designs in which the coil elements are segmented and connected by capacitors in series.

Experimental Setup and Results

The MTL model was applied to simulate the resonant spectrum of a 12-element high-pass birdcage coil having the following dimensions: d=30cm, diameter 8cm, shield diameter 12cm. The coil was constructed on an acrylic former using 6.4mm-wide strips of self-adhesive copper tape, and at both end rings 10pF nominal capacitances matched to better than 0.2% were used. Mutual and self-inductances of the longitudinal elements were calculated using the method of images, while the inductance of the end-ring segments was determined from the measured resonant frequency of the end-ring modes (406MHz). Coupling between end rings and between end ring segments was neglected as justified by the close proximity of the coil to the shield. Coil excitation was achieved using a small inductive probe connected to a network analyzer (HP 8753) and no direct connections to the coil were made. The frequency (384MHz) of the mode of order 0 (having currents of equal amplitude *and* phase on all longitudinal elements) was used to calculate the stray capacitance (0.47pF) between each end-ring segment and the shield. Detection of the *electric* field between the end ring and shield using a short dipole was required for this mode. Resonant frequencies of the remaining modes (azimuthal orders 1–6) were measured inductively and a second set of measurements was obtained with capacitors removed from one of the coil's end rings. A full resonant spectrum was observed nevertheless due to the parasitic capacitance between adjacent end-ring segments, which was estimated using the MTL model to be 0.7pF. The end-ring capacitance value that best fit the two sets of measured frequencies was 10.3pF, well within the capacitors' nominal 10% tolerance. The corresponding lumped-element model used for comparison utilized the same electrical parameters as the MTL model and the lumped inductances of the longitudinal elements were obtained from their distributed counterparts by multiplication by the coil's length. Figure 1 shows mode frequency as a function of mode order for the two capacitor configurations.

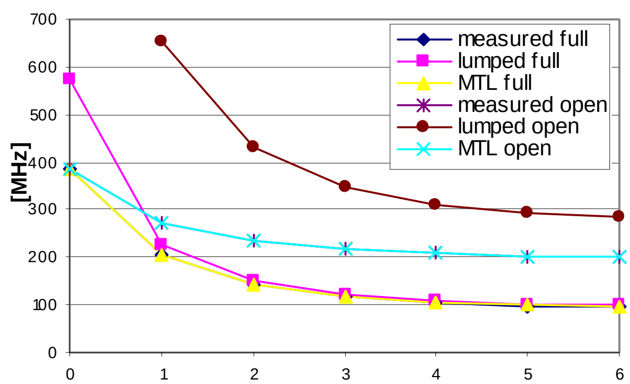

Figure 1: calculated and measured resonant frequencies vs. mode order of the 12-element birdcage coil. "Open" indicates the absence of end-ring capacitors at one end.

Conclusion

The MTL model demonstrates an improved ability to predict the resonant frequencies of the birdcage coil over the lumped-element model, particularly in situations where the distributed electric coupling between coil elements becomes significant, e.g., at high frequencies or when lumped capacitances are small. By being computationally advantageous over numerical methods, the multiconductor transmission line model can be a practical additional tool for the design and construction of birdcage coils.

Acknowledgements

We acknowledge financial support from the Alberta Heritage Foundation for Medical Research, and Natural Sciences and Engineering Research Council (Canada).

References

1. Doty et al. JMR 138, 144-154 (1999). 2. Foo et al. MRM 21, 165-177 (1991). 3. Joseph et al. JMR 75, 199-212 (1987). 4. Harpen. MRM 30, 489-493 (1993). 5. Baertlein et al. IEEE Trans. Biomed. Eng. 47, 535-546 (2000). 6. Bogdanov et al. MRM 47, 579-593 (2002). 7. Leifer. JMR 124, 51-60 (1997).