Classification of Fibre Tracts Using Differential Geometry

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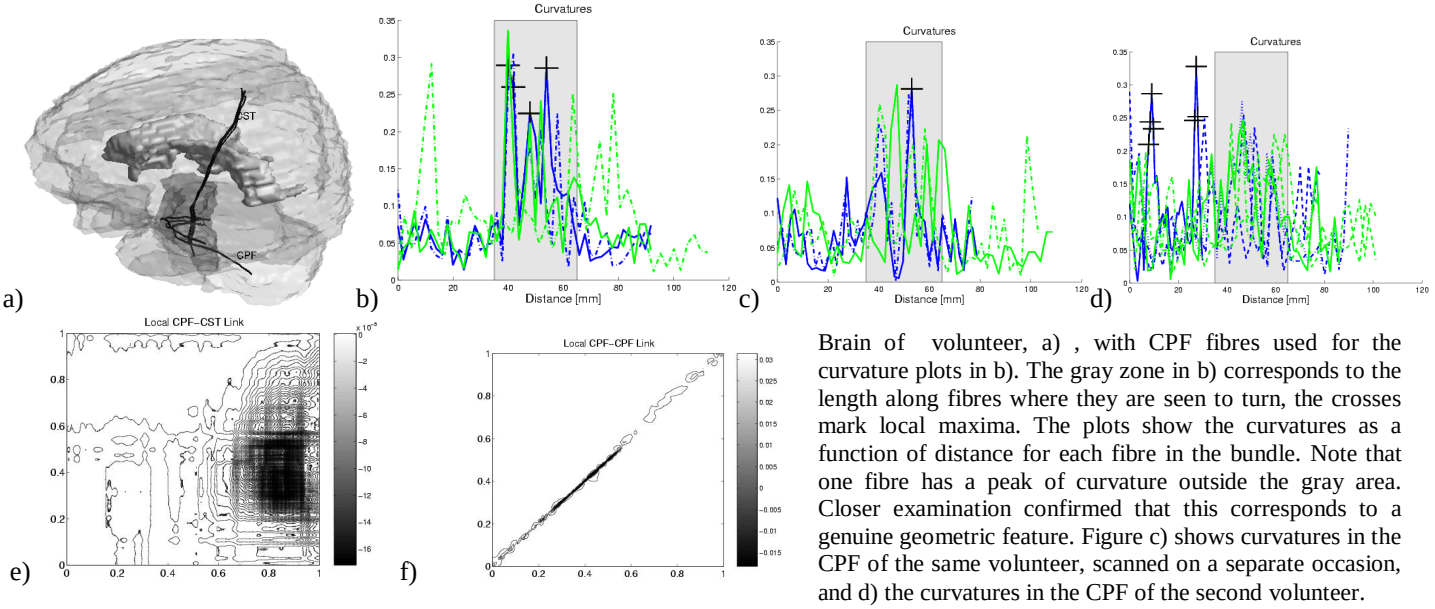
Introduction. Most fibre tracking studies in DT-MRI simply display the reconstructed fibres, and no information is usually calculated from them, which limits their clinical use. One of the main reasons for the limitation is that, while the degree of anisotropy of the tensors can be measured, the shape of the tracts is not directly quantified. Current fibre tracking algorithms produce fibres as spatial curves in three-dimensional space [1]. We propose to use tools from Differential Geometry to quantify the shape of fibres. These tools will allow intersubject comparison of individual curves, without the need for image normalisation. We also extend the geometric tools to the relative spatial configurations of curves/fibres, describing for example how spatial curves wind around each other, using the Link of a pair of curves, inspired from DNA and polymer folding analyses [2]. We constructed simulations to check the method (data not shown), and apply this method to three *in vivo* datasets.

Methods. We used 3 datasets from 2 volunteers, acquired using 20 gradient directions on a Siemens 1.5T scanner. Fibre tracking was performed using a standard streamline method (see e.g. [1]). The bundles used for investigating the method were the Cross Pontine Fibres (CPF) and Cortico Spinal Tracts (CST). The Frenet equations define the curvatures κ and torsions τ of a curve in 3D space (see [3] for definitions).The curvatures and torsions are parametrisation independent, and can thus be computed from any parametrisation of the fibre, e.g. by arc-length s. The curvature is then the magnitude of the second derivative, and the torsion involves third derivatives, measuring the speed at which the curve goes out of its tangential plane. The link (global and local) of a pair of curves c_1 and c_2 is defined below, where ' denotes derivative in the parameter s_1 or s_2.

$$L_{global} = \int_{c_2}\int_{c_1} \frac{c_1(s_1)-c_2(s_2)}{\left|c_1(s_1)-c_2(s_2)\right|^{(3/2)}} * (c'_1(s_1)\times c'_2(s_2))ds_1 ds_2 = \int\int L_{loc}(s_1,s_2)ds_1 ds_2 \,.$$

Results.



Brain of volunteer, a) , with CPF fibres used for the curvature plots in b). The gray zone in b) corresponds to the length along fibres where they are seen to turn, the crosses mark local maxima. The plots show the curvatures as a function of distance for each fibre in the bundle. Note that one fibre has a peak of curvature outside the gray area. Closer examination confirmed that this corresponds to a genuine geometric feature. Figure c) shows curvatures in the CPF of the same volunteer, scanned on a separate occasion, and d) the curvatures in the CPF of the second volunteer.

It can be seen that the fibres in the brain of the second volunteer have a different shape, with more curvature early along the fibre. e) and f) show the local link for one CPF and one CST fibre e) and a pair of CPF fibres, as a function of parameter along the curve (here normalised arc-length), represented as contour plots. The region where the CPF turns around the CST is clearly apparent in e), where f) shows that the two CPF curves follow approximately the same path.

Discussion-Conclusion. These tools, the curvatures and torsions of the curves in particular, allow statements to be made about the shapes of curves that are *independent* of their spatial position. This formalism could be used to quantify normal and abnormal tract shapes, for the classification of different fibre tracts, the characterisation of tracts that pass close to each other, and identification of functionally similar brain regions by examining the end points of tracts with similar shapes (see [4] for an early suggestion of using the tools proposed here, and [5,6] for related applications).

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