Application of the Laplace Transform to analyze intravoxel fiber structures in diffusion MRI

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Synopsis

The Laplace transform (*LT*) has been used to analyze fiber bundles in a voxel by applying diffusion MRI theoretically. From the *LT* results, it is possible to determine the number of fibers in a given voxel. The *LT* method can be thought as a complimentary method to HARD (high angular resolution diffusion-weighted) acquisition method to detect the fiber distribution in a voxel.

Introduction

Fiber tracking holds great promise as a technique to determine the orientation and connectivity of white matter fibers and anatomical connections between functionally active regions, for example. However, the detection of fiber-crossing is a challenging issue in diffusion MRI. The *LT* results of the diffusion MR signal provide an effective method to determine the number of fibers in a given voxel.

For a given encoding gradient $\mathbf{g} = [G_x, G_y, G_z]/|\mathbf{G}|$, the signal intensity in one voxel can be expressed as [1,2]

$$S / S_0 = \exp(-b \cdot ADC) \tag{1}$$

where *b* denotes the so-called *b*-value and S_0 is the signal intensity of reference image. In the following, we use the notation $D = ADC$ and $s(b,\boldsymbol{g}) = S/S_0$ which expresses the pure diffusion MR signal for the given *b*-value and the direction of diffusion encoding gradient, $\boldsymbol{g}$.

Theory

From the complex structure of brain tissue it is known that a given white matter voxel, for example, may contain many fiber bundles. The different fibers in the voxel may be described by different diffusion tensors. The fiber direction is assumed to be the same as the direction of the principle eigenvector. The contribution of each fiber to the measured diffusion MR signal can be described by a mono-exponential function as in Eq.(1). The measured diffusion MR signal from a given voxel is then the superposition of the signals from all fibers with different mono-exponential decays and is given by:

$$s(b, \boldsymbol{g}) = \sum_{k=1}^{N} P_k \exp(-bD_k) , \tag{2}$$

where P_k is the volume fraction and satisfies the normalization condition $\sum P_k = 1$. Eq.(1) describes the signal attenuation for a single resonance. Eq.(2) is generalized to represent the sum of the spectra from all components in a voxel. The influence of noise, which is present in measured diffusion MR data, has been neglected.

The *LT* method which can be used to represent the signal intensity, $f(t)$, for $t > 0$. *LT* is defined as:

$$L[f(t), p] = \int_0^{+\infty} f(t) \exp(-pt) dt \tag{3}$$

Indeed, it has already been shown that the inverse *LT* of $s(b,\boldsymbol{g})$ was implemented successfully in the DOSY method [3] to analyse the compartments of solution. In present work, $s(b,\boldsymbol{g})$ is the *LT* of P_k with respect to the *b*-value (see Eq. 2). The volume fraction P_k can be obtained using *CONTIN* and *DISCRETE* [4].

Results and Discussions

By way of demonstration, we investigated two cases. Case (*A*): One voxel contains three white matter fiber bundles, *fiber*1*a*, *fiber*1*b* and *fiber*1*c*, and they cross each other (Fig. 1a); case (*B*): Two white matter fiber bundles *fiber*1*a* and *fiber*2*a*, which are parallel to each other (Fig 2a). The laboratory reference frame is adopted. The fibers *fiber*1*a*, *fiber*1*b* and *fiber*1*c* have same eigenvalues (1.4, 0.35, 0.35) (unit: 10^{-3} mm²/s) but along different directions. The fiber *fiber*2*a* has eigenvalues (1.2, 0.60, 0.30) (unit: 10^{-3} mm²/s). The diffusion tensors and fiber directions are listed in Table 1. For a given direction of the diffusion gradient $= [g_X, g_Y, g_Z]$, the *LT* result of $s(b,\boldsymbol{g}) = \exp(-b \cdot ADC)$ with respect to the *b*-value gives $L[s(b,\boldsymbol{g}), p] = (1 + ADC)^{-1}$. In theory, the *LT* result of the signal intensity for cases (*A*) and (*B*) can be calculated by using Eq.(3). The *LT* results for cases (*A*) and (*B*) are presented in Figs.1b and 2b for the diffusion-encoding gradient direction $\boldsymbol{g} = [1, 0, +0.5]$. The coordinates of discontinuity give the *ADC* value D_k. For a continuous distribution as in Eq.(3), the peaks of *LT* of the diffusion MR signal can be regarded as corresponding to the fiber bundles. The number of peaks indicates how many main fiber bundles/trunks were present in the voxel. An automatic data analysis of the discrete components of the multi-exponential decay data can be performed by *DISCRETE* and *CONTIN*. For two parallel fibers of different diffusion tensors, the *LT* method can distinguish them in principle. From this point, LT method can be thought as a complimentary method to HARD acquisition method which cannot distinguish parallel fibers or more closely aligned fibers [5,6].

Table 1: Diffusion tensors and the directions of the different fibers.

Fibers	$(D_{xx}, D_{yy}, D_{zz}, D_{xy}, D_{xz}, D_{yz})$ *(10^{-3} mm²/s)*	*Directions of fibers*
*fiber*1*a*	(1.4, 0.35, 0.35, 0, 0, 0)	[1, 0, 0]
*fiber*1*b*	(0.35, 0.35, 1.4, 0, 0, 0)	[0, 0, 1]
*fiber*1*c*	(0.35, 0.875, 0.875, 0, 0, -525)	[0, 0.707, 0.707]
*fiber*2*a*	(1.2, 0.6, 0.3, 0, 0, 0)	[1, 0, 0]

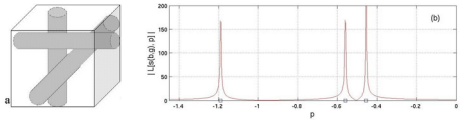

Fig.1 (a) Schematic representation of three fibers crossing each other in one voxel in case (A), and (b) *LT* results of diffusion MR signals in case (A) with the gradient direction $\boldsymbol{g} = [1, 0, +0.5]$.

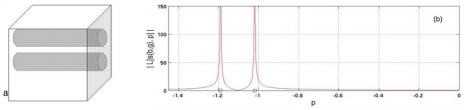

Fig.2 Same as in Fig.1 but for case (B). Two fibers parallel to each other in one voxel.

Acknowledgment: The authors thank Thomas Dierkes and Tony Stoecker for their discussions.

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