A Continuous Function Approximation of Scattered Data on the Sphere: Application to Diffusion Tensor MRI

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Introduction

There is an increasing interest in acquiring diffusion tensor (DT) MRI data with high angular resolution, using large number of different DW gradient directions and constant b-value. Such data improve estimation of the DT [1] and may also resolve multiple intravoxel DTs [2]. For these DT acquisition schemes, the DW signal depends only on the DW gradient direction, which is represented by a point on the surface of a unity-radius sphere. Thus, for any voxel, the DW signals are samples of a function, which varies on the spherical surface. This work has explored a method for approximating functions sampled on the spherical surface, and has used it in the analysis of DT MRI data from human brain.

Methods

Approximation technique Bicubic spherical splines [3] were used to generate a continuous function $S(\boldsymbol{g})$ (S: DW signal magnitude, $\boldsymbol{g}$: unity-magnitude vector of DW gradient direction), using the measured DW signals S_i, which correspond to DW gradient directions $\boldsymbol{g}_i$ (i=1,2,...,N, N is the total number of DW directions). The technique (algorithm SPHERE available from http://www.netlib.org) performs piecewise spherical harmonic transform (SHT), and thus can accommodate noisy data better than standard global SHT methods. $S(\boldsymbol{g})$ is determined after the sampled data have been smoothed by the minimum possible amount, which ensures continuity of the splines and their derivatives across neighbouring segments.

Application in PCA Recently [4] it was shown that principal component analysis (PCA) was able to determine principal diffusion directions at the same level of accuracy and precision as the standard analysis method of DT-MRI data (multivariate monoexponential fitting). However, the principal components (PCs) differed from the DT principal diffusivities. Thus we have used the above algorithm in order to determine DW signal magnitudes S_{pi} along the 3 PCA-derived principal diffusion directions $\boldsymbol{g}_{ip}$ ($S_{pi} \equiv S(\boldsymbol{g}_{ip})$, i=1,2,3). Principal diffusivities λ_{ip} are then calculated as $\lambda_{ip} = -\ln(S_{pi}/S_0)/b$.

Application for noise reduction Due to the smoothing operation of the algorithm, $S(\boldsymbol{g})$ will suppress random (noise-induced) variation among S_i. Such variation normally increases apparent anisotropy in areas of low anisotropy (for example cortical grey matter) and decreases contrast between low and high anisotropy in the DT anisotropy maps. Thus, the algorithm is used as a preprocessing step of the DW data before calculation of the DT. Specifically, the originally acquired samples S_i, are used only to determine $S(\boldsymbol{g})$ and then they are replaced by their approximations $S(\boldsymbol{g}_i)$, which are used as input to the DT fitting routine.

Evaluation The method was evaluated in simulations and human brain DT-MRI at 1.5T (Infinion, Philips Medical Systems, Cleveland, OH). The DT-MRI scheme used b=1200s/mm^2, 72 uniform DW gradient directions and 6 baseline signals (b=0). The simulations used SNR of the baseline signal 25, typical of that in white matter in our human brain DT-MRI datasets. Single-shot DW spin-echo EPI was used.

Results & Discussion

Table1

	$\lambda_1=\lambda_2=\lambda_3=0.7$			$\lambda_1=1.3\ \lambda_2=\lambda_3=0.4$			$\lambda_1=1.8\ \lambda_2=\lambda_3=0.1$		
FA	0.007±0.017	0.053± 0.017	0.007± 0.019	0.621±0.054	0.629± 0.049	0.633± 0.058	0.943±0.010	0.944± 0.009	0.942± 0.010
MD	0.697±0.016	0.697±0.016	0.697±0.016	0.693±0.018	0.692±0.016	0.688±0.016	0.693±0.018	0.695±0.018	0.686±0.027

Table 1 shows simulation results for three types of cylindrical fibres. For each type, mean value ± standard deviation are shown for fractional anisotropy (FA) and mean diffusivity (MD, in 10^{-3}mm^2/s). Left and middle columns show results for standard DT-MRI analysis with and without the proposed data preprocessing, respectively. Using the approximated data $S(\boldsymbol{g}_i)$ instead of the raw data S_i drastically reduces noise induced bias at the low anisotropy end, and does not affect estimation of true anisotropy. The right column under each fibre type shows results from the combination of PCA with the proposed method. Due to this combination, absolute quantification of diffusion properties is possible without the need for an explicit mathematical model of the DW signal (such as that used by the standard DT-MRI analysis method).

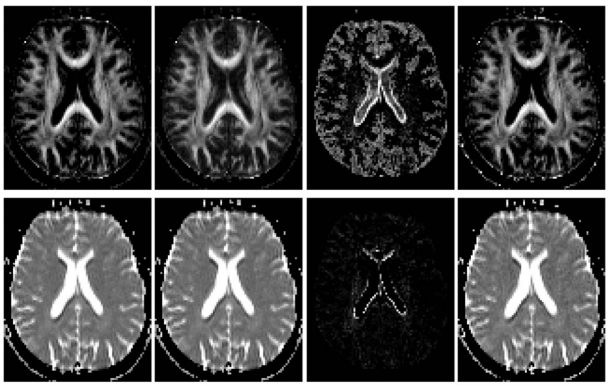

Figure 1

These observations are confirmed by Figure 1, which shows FA (top, scaled 0-1) and MD (bottom, scaled 0-1.6x10^{-3}mm^2/s) maps. Columns (a) and (b) show maps from the standard DT-MRI analysis with and without the data preprocessing, respectively. The difference (b)-(a) is displayed on (c). For both FA and MD it is positive, and is scaled 0-0.2 (FA) and 0-0.04x10^{-3}mm^2/s (MD). It should be noted that the preprocessing suppresses anisotropy at low anisotropy areas (within and around sulci and ventricles). The maps from the combination of PCA with the proposed method are given on (d), and are in agreement with the maps in (a), (b).

Conclusions

A method for approximating functions sampled on the sphere has been used in the analysis of high-angular resolution DT MRI data. Its application for data preprocessing suppressed noise-induced elevated anisotropy. Its combination with PCA enabled absolute quantification of diffusion properties, without the use of an explicit mathematical diffusion model. Ongoing work investigates the potential of the method to be used for the detection of multiple intravoxel DTs.

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