Modelling noise-induced fibre-orientation error

P. A. Cook[1], G. J. Parker[2], D. C. Alexander[1]

[1]Department of Computer Science, University College London, London, United Kingdom, [2]Department of Imaging Science & Biomedical Engineering, University of Manchester, Manchester, United Kingdom

Introduction. The Probabilistic Index of Connectivity (PICo)[1,2] framework uses Monte-Carlo streamline generation to create maps of connection probability. For each Monte-Carlo iteration, the primary eigenvector $\mathbf{e_1}$ of each tensor is aligned with a sample from the voxel Probability Density Function (PDF), and streamline tractography provides a sample fibre tract. After N iterations, the probability of connection is proportional to the number of streamlines that cross through each voxel.

For the case of cylindrically symmetric tensors, two PDFs have been used previously [1,2]. Both are one-dimensional normal distributions, where the deflection angle of the sample axis from the mean vector is the dependent variable and the standard deviation σ of the distribution in $\mathbf{e}_1$ as a function of the Fractional Anisotropy (FA) is used to parameterise the PDF [1,2]. The mean directions of the distributions are coincident with experimentally measured $\mathbf{e}_1$. However, these PDFs apply one-dimensional statistical techniques to a spherical sample space, necessitating *ad hoc* adjustments such as truncating the PDF to allow a maximum deflection of ±90°. Established parametric PDFs in spherical statistics provide more rigorous basis for work with axial distributions. The Watson distribution [3] describes the probability distribution for an axis coincident with the unit vector $\pm\mathbf{x}$: $f(\pm\mathbf{x}, \mu, \kappa) = M(0.5, 1.5, \kappa)^{-1} \exp[\kappa(\mu^T\mathbf{x})^2]$, where M is a Kummer function, μ is the mean of the distribution, and κ is a concentration parameter. For $\kappa > 0$, the distribution is bipolar with peaks at $\pm\mu$ and rotational symmetry about the axis $\pm\mu$. When $\kappa < 0$, the distribution is girdle shaped, and symmetric about the equator. This can be used to model the case of oblate tensors, where $\mathbf{e}_1$ is localized to a plane normal to the third eigenvector. The Watson PDF is continuous, and requires no truncation of the range of samples. We built a lookup table of κ, indexed by the two largest eigenvalues normalised by the smallest. We compute each κ in the lookup table by finding the maximum likelihood estimate from 10,000 samples for each tensor shape.

Sample probability maps from brain data are shown in [1] and [2], but in brain data there is no gold standard of anatomical connectivity by which to validate the results. We show PICo probability maps generated from synthetic data, where the actual connectivity is known and we have defined a gold standard of the uncertainty due to signal noise. The Watson PDF and the PDF from [2] correlate well with the gold standard maps.

Methods. We first describe experiments on synthetic data, generated by emulating the scanner sequence and assuming a signal to noise ratio of 16 in the b = 0 images. The tensor used to generate the noise free signal was cylindrically symmetric, with a trace of 2100×10^{-6} mm^{2}s^{-1}. We tested tensors at (pre-noise) Fractional Anisotropy (FA) values between 0.2 and 0.8. For the Gaussian PDFs σ was calculated from the functions used in [1,2].

Results

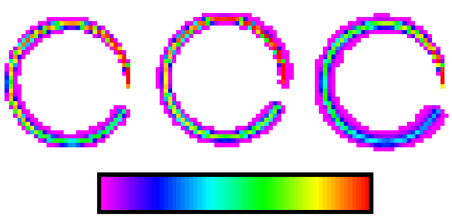

Figure 1: PICo maps in synthetic data using the Watson (left), gold standard (centre) and the Gaussian PDF in [2] (right)

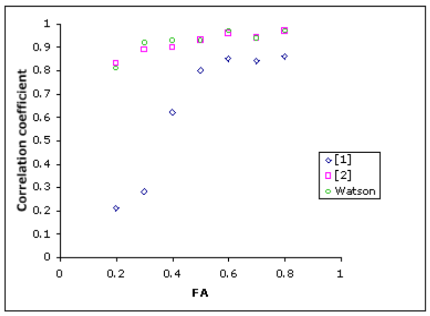

Figure 2: Correlation of each PDF to the gold standard probability map

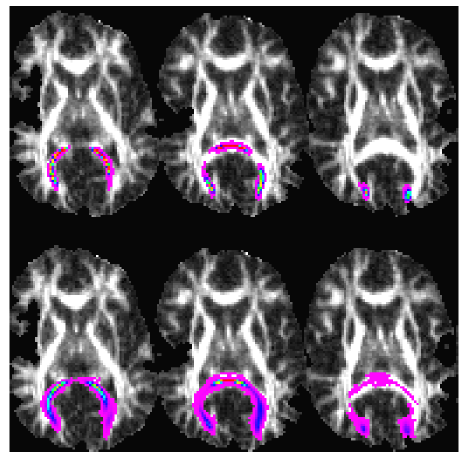

Figure 3: PICo probability map in the corpus callosum: Watson (top), Gaussian PDF [1] (bottom)

We computed PICo probability maps with each PDF by tracking counter-clockwise from the end of a circular path (radius 20mm, thickness 10mm, see figure 1), where each $\mathbf{e}_1$ was aligned with the tangent to the circle, and N = 10,000. We repeated this process 10 times with a different noise realization in the volume. As a gold standard probability map, we used simulated noise for the Monte-Carlo reorientation of the tensors. Instead of sampling a PDF in each voxel, we added noise to the complex MR signal and re-fitted the tensor. Figure 2 shows the correlations between the gold standard and the maps generated with each PDF.

The PDF from [1] gave a much less focused probability map than the other two methods. The difference this makes in brain data can be seen in figure 3. The authors of [1] intended their PDF to model total uncertainty in fibre orientation, not just the effect of noise. When FA is low, they argue, uncertainty is high because of many factors, such as partial volume effects. However, the function they use is heuristic and no justification is given for its choice. In [2], the PDF is designed to model noise, and the value of σ was fitted to simulated distributions, in a similar way to our lookup table. The correlation of PDF [2] to the gold standard is not significantly different from the Watson. Both perform less well at low FA, producing a wider dispersion of streamlines than the gold standard simulation. Underestimation of κ due to outliers is a known problem with spherical distributions, and we are currently investigating methods to improve our lookup table of κ.

The Watson PDF should be preferred to the Gaussian PDF because it is continuous, integrates to unity, and can be used with negative κ to model the uncertainty in oblate tensors. Furthermore, statistical methods exist to estimate both the concentration and mean axis of a set of sample axes. If spherical models were used in future studies the estimated parameters could be tested within the PICo framework. The Watson model also extends naturally (via the Bingham distribution) into a PDF for the non-symmetric case, which will capture the uncertainty *in vivo* more accurately, and can further be applied to multifibre decomposition [2].

Acknowledgement. We would like to thank Olga Ciccarelli and Claudia Wheeler-Kingshott of the Institute of Neurology, University College London, for providing the human brain data. This work was funded by the MRC/EPSRC MIAS-IRC under grant GR/N14248/01.

References

1. GJM Parker, et al, *Journal Of Magnetic Resonance Imaging* 18:242-254 (2003)
2. GJM Parker and D C Alexander, IPMI 2003, LNCS 2732, pp 684-695.
3. GS Watson, *Biometrika* 52(1):193-201 (1965)