RESTORE: Robust Estimation of Tensors by Outlier Rejection

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INTRODUCTION: Signal variability in diffusion weighted imaging (DWI) is affected by “background noise” and “artifact-related noise”. Artifact-related noise includes for example, subject motion, cardiac pulsation, and spike noise. While background noise is approximately Gaussian distributed [1], artifact-related noise does not have a known parametric distribution and currently cannot be modeled. The diffusion tensor is commonly estimated on a voxel-by-voxel basis by fitting the signal decay as a function of the b matrix, using the assumed signal variance as a weighting factor [2]. The signal variance due to background noise is constant for all points and can be accurately estimated in the background of the image [1]. This standard tensor fitting approach, however, neglects the contribution of artifact-related noise, which will produce outliers that can severely bias the estimated tensor. With few exceptions [3] the issue of robust estimation of the diffusion tensor has been largely neglected. This is surprising because artifact related noise is common in clinical DT-MRI acquisitions. An approach based on the Geman-McLure M-estimator minimizes the influence of outliers over the tensor estimation [3]. In this approximation, each data point is not equally weighted, and a weighted least squares (LS) method is used to estimate the parameters. Since the weights for good data points can be biased by the outliers during the iterative fitting, this approach may not be the most effective strategy against outliers. Here we propose an alternative approach, robust estimation of tensors by outlier rejection (RESTORE), to identify and remove the outliers using an iterative re-weighting process. Once the outliers are removed, each remaining data point is weighted equally and the tensor is re-estimated.

METHODS: The initial tensor estimation is computed by a non-linear least squares method with equal weights. The residuals are used to estimate the variability of the data points. A threshold criterion of three standard deviations of the expected signal (=1.5267*standard deviation of the background noise) [1] is chosen to determine if the data fits the model adequately. For every data point, if its residual is less than the threshold, the tensor estimation is obtained. Otherwise, an iterative re-weighting process begins. It assumes the reciprocal of the square of the residual from the previous fitting as the weight of that point. This process converges when there is no change in the current compared to the previous fitting. The outliers are identified and excluded using the residuals computed from the final re-weighting process and the same threshold criterion.
In our clinical DTI studies, four repeated acquisitions of seven images (i.e. 6 DWIs and 1 b=0 image) are typically performed. To simulate this, we created 28 synthetic DW images from a precomputed diffusion tensor volume and an assumed set of standard b-matrices [4] (with four replicates per b-matrix). We also added Gaussian distributed noise in quadrature so as to simulate images with a signal to noise ratio (in the b=0 image) of 25. We randomly selected one image out of the 28 and altered its intensities by a factor of ten either lower or higher than its original values.

RESULTS: In Fig. 1, we show the results of tensor estimation using (a) non-linear LS, (b) iterative re-weighting LS and (c) RESTORE methods, and compared to the (d) ground truth. It is noticeable that the color maps of (a) and (b) have a red hue when compared to (d), for example in the circled area. The corresponding difference map is shown in grey scale where the resulting pixel intensity is the absolute value of the difference from pixels in the two images, the tensor image obtained by one of the three methods and the ground truth. Qualitatively, a single corrupted image causes considerable errors in the tensor estimation for (a) non-linear LS and (b) iterative re-weighting LS in this simulation while the intensity of one image is altered by 0.1. To quantify the result, we calculate the relative error of the non-linear LS method, the iterative re-weighting LS method, and our RESTORE approach on both the trace and fractional anisotropy (FA) images. The relative error for the trace is defined in equation (1) where *N* is the total number of voxels in the image volume and the index *i* corresponds to each voxel in that set. A similar definition is applied to the FA map. Table 1 shows the average errors using these methods. It is evident that just one bad image degrades the FA map in the non-linear LS case, while the iterative re-weighting LS approach shows an improvement on the tensor estimation. Finally, the RESTORE approach shows a significant improvement over the other two methods.

CONCLUSION: Our study shows that the proposed method improves the quality of diffusion tensor mapping when outlier DWI data are present. Such outliers are frequently encountered in clinical diffusion-weighted scans, for example, with uncooperative subjects or in pediatric studies. The RESTORE method should therefore extend the clinical utility of DT-MRI and its usefulness in a number of clinical applications is currently under investigation.

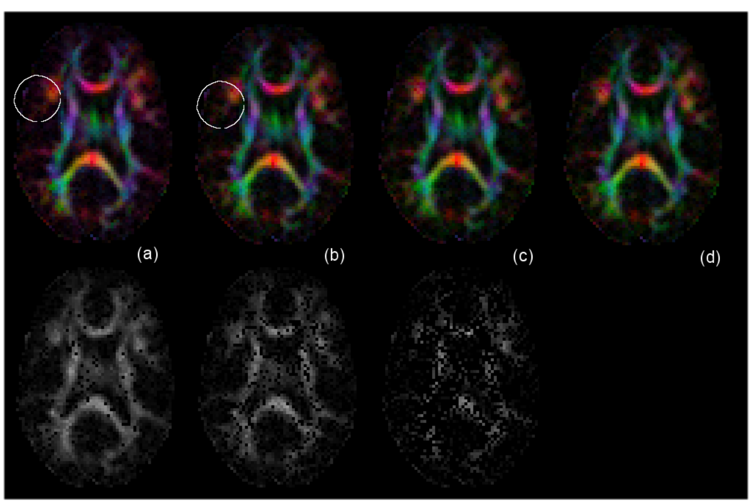

Fig. 1 The color maps obtained using (a) non-linear LS, (b) iterative re-weighting LS, (c) RESTORE and (d) the ground truth. The corresponding difference maps are shown as gray scale images where a higher intensity represents a stronger variation. (In this case, the intensity of one image is reduced by ten fold for simulation.)

$$Err_{trace} = \frac{1}{N}\sum_{i}\frac{|Trace_{corrupted(i)} - Trace_{original(i)}|}{Trace_{original(i)}} * 100 \qquad (1)$$

	one image with lower intensity (*.1)			one image with higher intensity (* 10.)		
%	non-linear	reweighting	robust	non-linear	reweighting	robust
Trace	5.0698	2.2014	0.8117	23.3618	1.7584	0.6166
FA	54.8132	20.9915	7.20556	410.552	16.4962	5.5854

Table 1. Average relative errors in percent

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