# The Effect of Gaussian Noise on Generalized Diffusion Tensor Imaging

C. Liu[1,2], R. Bammer[1], M. E. Moseley[1]

[1]Radiology, Stanford University, Stanford, CA, United States, [2]Electrical Engineering, Stanford University, Stanford, CA, United States

## INTRODUCTION

Recently a generalized diffusion tensor imaging (GDTI) method was introduced to characterize non-Gaussian diffusion (1,2). It has been shown that non-Gaussian properties of a diffusion process can be characterized by a series of generalized higher order diffusion tensors. Those higher order diffusion tensors (HOT) can be used to reconstruct the probability density function (PDF) of the random displacement. In this abstract, the effect of Gaussian noise on the HOTs and the reconstructed PDF is investigated by using random walk simulation. Gaussian noise with two levels of standard deviation was added to the signal. The estimated HOTs and the reconstructed PDF are compared to that of the noiseless situation.

## MATERIALS AND METHODS

Random walk simulation was performed on two phantoms: two perpendicularly crossing tubes (Phantom 1), and a Y-shaped tube (Phantom 2). All the tubes have a square cross section with a width of 20 µm. The ends of the tubes are closed. The boundaries of the tubes were assumed to be impermeable.

The particle motion is represented as a stochastic process, denoted by $\mathbf{x}(t)$. The process behaves in the interior of the phantoms like a standard Brownian motion. At the boundary $\mathbf{x}(t)$ reflects elastically. The particle motion is represented as a sequence of small random displacement: $\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \Delta\mathbf{x}$.. Here, $\Delta\mathbf{x}$ is the random displacement of the particle in the time interval $\Delta t$. $\Delta\mathbf{x}$ is generated according to the Gaussian distribution with zero mean and variance $2D\Delta t$. The particle trajectory is obtained by using the above equation. The trajectory is corrected with consideration of the boundary condition. For perfectly reflecting walls located at $x_i = -a$ and $x_i = a$ (here $i$ indicates the $i$-th coordinate), the boundary effect is taken into account by replacing $x_i$ with when $-2a - x_i$, or by replacing $x_i$ with $2a - x_i$ when $x_i \geq a$.

For a spin echo sequence, the simulated spin phase at TE for a single spin $n$ is given by (3)

$$\Phi_n = \gamma \mathbf{G}\cdot\left(\sum_{t=0}^{\delta}\mathbf{x}_n(t) - \sum_{t=\Delta}^{\delta+\Delta}\mathbf{x}_n(t)\right)\Delta t \quad , \qquad [1]$$

where $\mathbf{x}_n(t)$ is the position vector of spin $n$ at time $t$. In this simulation, we consider only the central voxel of size 80x80x80 $\mu m^3$. The signal at TE is obtained by summing the magnetization of all the spins located in this voxel. The generalized higher order diffusion tensors were estimated according to the following signal equation (1, 2),

$$m(b) = m(0)\exp\left(\sum_{n=2}^{\infty} j^n D^{(n)}_{i_1 i_2 \Lambda\, i_n} b^{(n)}_{i_1 i_2 \Lambda\, i_n}\right). \qquad [2]$$

A total of $2.5\times10^5$ spin trajectories with uniformly distributed starting positions were simulated. The following parameters were used: $\Delta t = 0.2$ ms, $\delta = 30.0$ ms, $\Delta = 40.0$ ms, TE = 80.0 ms, $G_{max} = 40$ mT/m, and $D = 2.02 \times 10^{-3}$ mm$^2$/s. Gaussian noise with different variance is added to the signal in order to assess the effect of noise. The PDF can be computed by using the Gram-Charlier series (1, 2).

## RESULTS

Fig. 1 shows the reconstructed PDF skewness map and iso-surface map for both Phantom 1 and 2. Here red indicates negative value and green represents positive value. Column (a) is the result in the absence of noise. Column (b) and (c) are results obtained under two different noise levels. The percentage error in the estimated tensor elements introduced by the additive Gaussian noise is given in table 1.

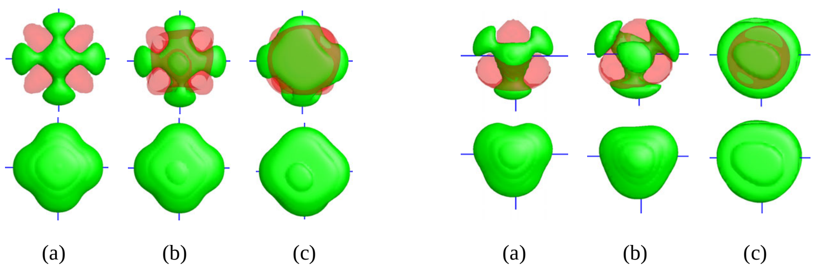


**Fig. 1** – The three columns on the left are results for Phantom 1. The other three columns on the right are for Phantom 2. The first row is skewness map and the second row is PDF iso-surface. (a) noiseless; (b) SNR = 31.5 when b = 313 s/mm$^2$, and SNR = 4.6 when b = 3126 s/mm$^2$; (c) SNR = 15.9 when b = 313 s/mm$^2$, and SNR = 2.4 when b = 3126 s/mm$^2$.

| % error | | $D^{(2)}_{i_1 i_2}$ | $D^{(3)}_{i_1 i_2 i_3}$ | $D^{(4)}_{i_1 i_2 i_3 i_4}$ |
|---|---|---|---|---|
| Phantom 1 | SNR (b) | 1.1% | N/A | 2.2% |
| | SNR (c) | 3.0% | N/A | 6.9% |
| Phantom 2 | SNR (b) | 2.4% | 12% | 8.6% |
| | SNR (c) | 4.1% | 16% | 12% |

**Table 1** – Percentage of error of the measured HOT elements. For each tensor-order, the error is computed as the average of the absolute value of the errors over all the elements.

## DISCUSSION

When the deviation from a Gaussian distribution is not large, the magnitude of the higher order statistics of the PDF will be small. In general, higher order tensor elements are more susceptible to noise. As seen in table 1, the percentage error increases for tensors of higher rank. This is consistent with the fact that it is more difficult to measure the higher order moments of a random variable. In particular, tensors of odd order appear to have higher percentage of error than tensors of even order. For phantom 1, the percentage of error of the 3rd order tensor is not computed because the 3rd order tensor is zero for a symmetric PDF. When the PDF is reconstructed by using the HOTs measured in the noisy situation, its quality will be degraded as illustrated in Fig. 1. Nevertheless, for a reasonably high SNR, for example, when SNR = 4.6 at b = 3126 s/mm$^2$, the PDF is not significantly distorted. Other errors can be introduced in an *in vivo* experiment besides that from Gaussian noise. Further study is needed to characterize other forms of error.

## ACKNOWLEDGMENTS

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