A Novel Denoising Technique for very Noisy Diffusion Tensor Imaging Data

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Introduction Experiments with low signal-to-noise ratios (SNR), e.g. measurements with small voxels or high b values, convey essential information about anisotropy and fiber organization in soft and deep white matter tissue. However, due to dominating and complicated impact of thermal noise, such data are still seldom analysed. Voxelwise denoising, by replications of minimal experiments or by multigradient experiments, often cannot enlarge SNR sufficiently, as e.g. tracking is highly sensitive to residual noise **[1].** As will be shown, convenient coupling with spatial filtering improves the situation essentially, reducing too high numbers of replications or scanning times. To optimise filtering, peculiarities of diffusion tensor imaging (DT-MRI) data must be incorporated. Also, the variables with advantageous noise distributions for filtering must be found. In this context, by Monte Carlo simulations, the random fields of DTI variables are probed. For convenient variables a new three dimensional edge preserving filter is proposed and validated by a "gold standard" model. The algorithm estimates local mean values, works fully automatic (the only data driven input is background noise level), is fast and statistically robust also in case of skewed noise. An application of the method to data from a very recent experiment with 1x1x1 mm^3 voxels is finally presented.

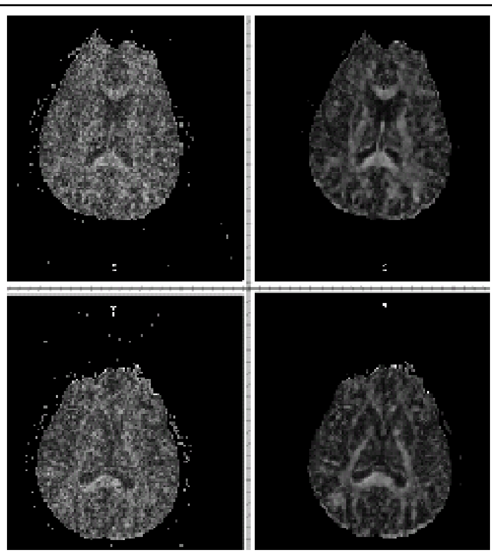
Methods and Materials The essential input for the Monte Carlo simulations (MCs) were : a) cigar shaped, diffusion model, b) Stejskal-Tanner equations for low and high b value experiments with minimal- and multigradient arrangements, c) different high noise levels applied to the signals. The distributions for the variables: diffusion weighted images (DWI) or magnitude signals, tensor, eigenvalues, tensor trace, fractional anisotropy and main eigenvector orientation were investigated by their relative bias, bias uncertainty, skewness and heteroscedasticity (het). The SNR range investigated was between 1 and 20. The proposed spatial filter consists of a chain of nonlinear three dimensional Gaussian filters estimating local mean values of scalar fields, f(x). Formally, this construction is given by

$$f_k^{smooth}(x) = F_{\eta^* c^{k-1}}^{\mu^* d^{k-1}} \circ \ldots \circ F_{\eta^* c}^{\mu^* d} \circ F_{\eta}^{\mu} \circ f(x) \quad \text{with} \quad F_{\eta}^{\mu} \circ f(x) = \sum_{y \in \text{Neighborhood of } x} \Phi(x,y)\Psi(f(x), f(y)) f(y) \Big/ \sum_{y \in \text{Neighborhood of } x} \Phi(x,y)\Psi(f(x), f(y))$$

The filter can be modified to smooth weakly non-Gaussian random fields, varying spatially in the brain, when variance and bias can be parametrized (M1). To reduce blurring, the filter is edge preserving, as anisotropy and fiber directions can change within very few voxels. A different edge preserving filter was applied to DTI by Parker et al. **[2].** Edges marginate architectural units in white matter, like fiber bundles, and do not appear e.g. in every DWI; their localisation, however, can be incorporated into the nonlinear filter windows,Ψ. Similarly, the shape of the diffusion ellipsoid can be incorporated into the linear spatial windows, Φ, for uniform fiber bundles (M2), see **[3]** for more details. To test the applicability of the presented theory to very small voxel sizes, isotropic 1 mm^3 diffusion weighted data were acquired from a consented normal volunteer (F40y) on a General Electric 1.5 MRI scanner using a dual spin echo prepared diffusion sequence that utilized ramp sampling and fat suppression. The tensor encoding scheme used is the principal icosahedral (Icosa6) see **[4]**. The field of view is 260 mm^2 and the data matrix is 256x256 pixels, a total of 28 contiguous axial sections covering the corpus callosum were selected from a sagittal scout localizer. The b factor is 1000 s mm^{-2}, TR=4.5 seconds and TE=82 ms and NEX=4. The magnitude constructed images were averaged by the scanner.

Results The MCs indicate that for all variants of minimal low SNR experiments (replications included) the DTI variables are distributed by random fields with strong local variation in the brain. DWIs offer the most convenient distributions for spatial filtering, as in most cases their deviations from uniform bell shaped densities is smallest, this is in line with conclusions in **[1]** for SNR>20. In addition, their Rician distributions offer a convenient parametrization for filter modifications. Multigradient experiments for low SNR reduce bias in all variables (except DWI) but skewness and het above all for the main diffusion directions. Filter applications to DWIs were validated by a three dimensional gold standard model (corpus callosum region) distorted by artificial Rician noise. The Table presents reduction of initial noise (σ) by a filter adapted to Rician noise (M1) and additional to DTI geometry (M2) for DWIs with gradients indicated. The last line gives the mean numbers of experimental replications with equivalent denoising effect. The Figures show axial FA maps around corpus callosum for the small voxel experiment, before and after spatial smoothing the DWIs. The mean SNR is approximately 3. Optimal filter performance is achieved for the isotropic voxels.

Discussion and Conclusion Voxelwise smoothing of replicated minimal experiments coupled with spatial filtering is a reasonable strategy for denoising low SNR data. In general, these methods should be applied to the DWIs. The spatial filter is as effective as n_F = 4 to 8 replications with DWI averaging, depending on the noise level. A coupling of both methods reduces noise by sqrt($n_F * n_{REPLIC}$). FA maps of very good quality can be achieved by spatial smoothing alone, the high sensitiveness of tracking **[1]** suggests spatial smoothing after DWI averaging; because, with noise reduction also filter blurring by averaging is reduced. See **[3]** for more details and thourough analysis.



	σ=60 σ_{M1}	σ=60 σ_{M2}	σ=90 σ_{M1}	σ=90 σ_{M2}	σ=120 σ_{M1}	σ=120 σ_{M2}
(1,0,0)	27	23	36	31	42	41
(0,1,0)	28	24	36	32	44	41
(0,0,1)	29	24	40	32	47	41
(1,1,0)	29	23	35	31	42	41
(1,0,1)	29	23	37	31	43	41
(0,1,1)	30	24	37	31	44	41
n_F	4.4	6.5	6	8	7.5	8.6

Fig. 1: Axial FA maps around corpus callosum for an experiment with 1 mm^3 voxels, see text. Left panels, for raw data, right panels, after spatial filtering.

Tab. 1 : Spatial mean deviations between model and noisy (σ) or filtered DWIs (σ_M) Last row gives mean numbers of experimental replications with equivalent denoising, else see text.

References

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