# Diffusion Anisotropy With And Without The Tensor Model

J. G. Pipe[1], K. Arfanakis[2]

[1]MRI Department, Barrow Neurological Institute, Phoenix, AZ, United States, [2]Department of Biomedical Engineering, Illinois Institute of Technology, Chicago, Illinois, United States

**INTRODUCTION:** Conventional fractional anisotropy (FA) calculations [1] from multi-angle diffusion measurements require fitting the data to a tensor model, and then calculating the metric FA based on the 3 eigenvalues $\{\lambda_i\}$, i.e.

$$FA = \sqrt{(\lambda_1-\lambda_2)^2+(\lambda_1-\lambda_3)^2+(\lambda_2-\lambda_3)^2} \Big/ \left[2\left(\lambda_1^2+\lambda_2^2+\lambda_3^2\right)\right].$$

It has been shown that for complex systems such as that produced by tracts oriented in different directions within a single voxel, the tensor model cannot reliably reflect the data if the number N of diffusion measurements is greater than six (i.e. the number of degrees of freedom supported by the tensor model), and information may be lost during the fit of the data to a tensor model (e.g. [2]). Another metric, G, for anisotropy was previously introduced and shown to be exactly equivalent to FA when data are measured in the 6 directions given by an icosahedral scheme [3]. Given N diffusion measurement $\{d_i\}$, defining $d_{trace} = \sum d_i / N$, $d_{rms} = \sqrt{\sum d_i^2 / N}$, and $d_{norm} = d_{trace}/d_{rms}$,

$$G = \sqrt{(3/2)\left(1 - d_{norm}^2\right) \Big/ \left(1 - \{3/5\}\, d_{norm}^2\right)}.$$

**METHODS:** In this work, using both simulations and collected data, G was compared to FA for icosahedral diffusion measurement schemes [4] with N=6, 12, and 30 directions, using b=1000 sec/mm$^2$. The number of b=0 images was kept at N/6. G and FA were compared using the original data $\{S_i\}$. Both simulated and collected data were also smoothed by taking $\{S_i\}$, calculating diffusion values $\{d_i\}$, fitting these to a tensor, projecting the tensor diffusion back onto the N measurement directions, and calculating a new tensor-consistent diffusion-weighted signal $\tilde{S}_i$ using the original b=0 signal ($\tilde{S}_0 = S_0$). G and FA were again compared using $\{\tilde{S}_i\}$. *Simulations*: Synthetic data from two tracts (eigenvalues 0.0017, 0.0002, and 0.0002 mm$^2$/sec: one fixed along the x-axis, the other rotated through 180˚ about the Z axis) were combined along with different levels of noise. In-Vivo *Data Collection*: Data were collected on a GE 1.5T scanner using a volunteer, with 2x2x3mm resolution over the whole head for N=6 and 12 diffusion measurement directions, keeping scan time constant (9 minutes). These scans were repeated 3 times (6 scans total) with the volunteer keeping his head still.

**RESULTS**: The following results are consistently seen in both simulations and in-vivo data. For N=6, G = FA (this was previously proven analytically). For N > 6, G ≥ FA. For synthesized (noiseless) data from single-tensor, and for any data $\{S_i\}$ smoothed by a tensor fit as described above, G = FA. Note that this last equality, while consistent, is not exact; at this time, it is thought (but not known) to be within the numerical precision of the calculations. An example of the results is shown in Fig. 1 below.

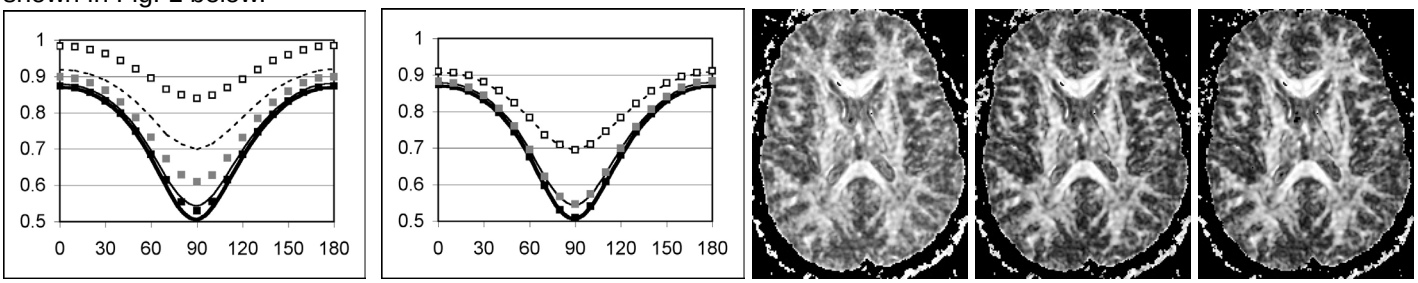


Fig. 1. Simulated N=12 data for (a) original and (b) tensor-smoothed data, showing FA for SNR=infinity (bold line), 20 (medium line), and 8 (dashed line) and G for SNR=infinity (black squares), 20 (gray squares), and 8 (open squares) (SNR is defined for the composite b=0 data). Images (identically windowed) are from a single N=12 scan, showing (c) G using original data, (d) FA using original data, and (e) G using tensor-smoothed data. Image (d) and (e) are visually identical.

**DISCUSSION**: The obtained results suggest that the differences between G and FA are from the anisotropy in the data that cannot be fit to a single tensor, whether that is from noise or from more complex (e.g. multi-compartmental) diffusion. This relationship may prove to be useful for analyzing the effect of the tensor model on FA in various situations. Furthermore, with high SNR data sets where the majority of the "residual" non-tensor diffusion anisotropy is from complex diffusion, G may be a preferable metric to FA.

**REFERENCES**: 1. Pierpaoli C, Basser PJ, Magn Reson Med 36(6), 893-906. 2. Frank, LR, Magn Reson Med 45(6), 935-959. 3. Pipe JG, Farthing VG, Magn Reson Med 49(3), 536-542. 4. Hasan KM, Parker DL, Alexander AL, JMRI 13(5), 769-780.