# Numerical analysis of the feasibility of finite-δ q-space probability measurements in human white matter

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**Introduction:** Diffusion MRI (DMRI) enables the study of water diffusion in a variety of environments (e.g. biological tissues). A common DMRI pulse sequence is the pulsed-gradient spin echo (PGSE) *(1)*. The relevant diffusion-encoding parameters are the strength and direction of the diffusion-encoding gradient **g** (applied before and after a 180° RF pulse), the angle θ between the direction of **g** and the axis of the cylinder, the time offset (Δ) between the start of the two encoding gradients, and the gradient duration ($\delta$). The PGSE voxel signal attenuation is $\langle e^{i\phi}\rangle_{\Delta,\delta,\mathbf{g}}$, where Δ, $\delta$, and **g** are fixed. In *(2)*, a q-space method was developed that would correctly account for the effects of a finite-duration encoding gradient on the water displacement probability measured by q-space imaging, for the case of Gaussian diffusion. The method was found to work for impermeable spherical and cubic domains provided those domains are not too small. In this abstract, we analyze the capacity of that altered q-space formalism to correctly describe the displacement probability in the case of impermeable (restricted) cylindrical domains.

**Theory and Methods:** ***Finite-δ q-space model.*** The DMRI signal attenuation can be visualized in several ways, and open different windows on the dynamics of the water molecules causing the dephasing phenomenon. Consider a water molecule moving between time *t=0* and the echo time, and having a spatial location $\mathbf{r}(t)$. In *(2)*, a q-space method was developed that would correctly take into account the effects of a finite-duration encoding gradient if the displacement probability distribution $P(\mathbf{r},t)$ is a linear combination of zero-centered Gaussians. In that formalism, the relation between signal attenuation and $P(\mathbf{r},t)$ is:

$$P(\mathbf{r},\Delta+\delta)=\int_{-\infty}^{\infty}\left\langle e^{i\phi}\right\rangle_{\Delta,\delta,\mathbf{g}}\cos(\gamma\delta\eta\mathbf{g}\cdot\mathbf{r})\,d(\gamma\delta\eta\mathbf{g}) \quad \text{with} \quad \eta=\sqrt{\frac{\Delta-\delta/3}{\Delta+\delta}} \quad . \qquad [1]$$

In Eq. 1, "$\Delta+\delta$" is the true diffusion time of the experiment *(2)*.

**Simulations:** In white matter, cell shapes resemble cylinders. Moreover, the permeability of axonal membranes is finite and close to zero ($P_{in->ex}\approx 0.003$ *(3)-(4)*). Therefore, Eq. [1] needs to be validated in the case of cylindrical restricted domains. In this simulation, a single domain of radius *a* is centered at the origin. Because the location of the starting point might in this case be very relevant, the starting point is a random variable that is uniformly distributed within the restricted domain. The number of simulated trajectories is 500 and the number of steps during the true diffusion time "$\Delta+\delta$=*60 ms*" is 500. The quality of the fit between the simulation results and the finite-$\delta$ q-space model are calculated for $\delta$ values ranging from 0 to 30 ms (Fig. 1). The time where Eq. [1] (middle blue curve in Fig. 1) is no longer obeyed is called the critical δ time (vertical black line in Fig. 1). We generated graphs of the critical time $\delta_c$ (Fig. 1) as a function of radius *a* ranging from 1 to 5 μm and as a function of angle θ ranging from 0 to 90 degrees (Fig. 2), where allowed settings of $\delta$ are under the black curve in Fig. 2.

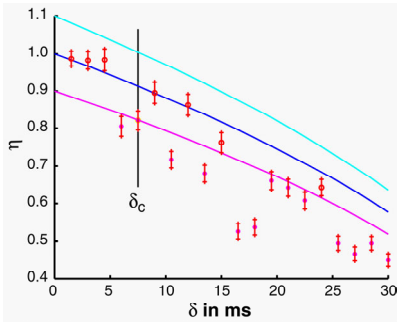


*Fig. 1: Graph of η as function of δ. Cyan and magenta lines are ± sd.*

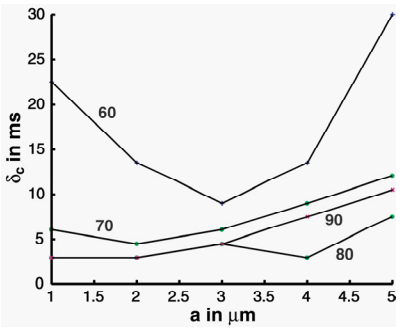


*Fig. 2: Graph of $\delta_c$ (Fig. 2) as a function of radius, for different values of θ.*

**Results:** For the impermeable cylindrical domains, we observed that the values of $\delta_c$ are well within practical values of human DMRI for θ<50° ($\delta_c \geq 25$ ms, not shown). The results for the other angles (θ>50°) can be seen in Fig.2. The allowed values of $\delta$ are those that obey "$\delta \leq \delta_c$".

**Discussion and Conclusion:** A revised formalism for measuring probability distributions from *q*-space data *(2)* is tested (Eq. [1]), in a white matter-like situation, using random walks to approximate the trajectories of water molecules *(5)*. The formalism is confirmed even when water molecules are constrained inside an impermeable cylindrical domain. The cylindrical domain must be sufficiently large (see Fig. 2), a requirement that is met by all axonal dimensions if θ is less than 50°. These results relax the infinitesimal-δ requirement of q-space MRI, which would otherwise severely limit this method in human white matter studies *(6)*. These results may also help validate q-space results in human white matter *(7,8)*.

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**References:** ***(1)*** Stejskal EO, Tanner JE, *J Chem Phys* 42:288-92, (1965); ***(2)*** Lori NF, Conturo TE, Le Bihan D, J Mag Reson, 165, pp. 185-95; (2003); ***(3)*** Meier C, Dreher W, Leibfritz D, *Magn Reson Med*; Vol. 50, pp. 500-9, (2003); ***(4)*** Meier C, Dreher W, Leibfritz D, *Magn Reson Med*; Vol. 50, pp. 510-4, (2003); ***(5)*** McQuarrie DA, Statistical mechanics. Sausalito, California: Univ. Sci. Books, (2000); ***(6)*** Basser P, *Magn Reson Med* 47:392-7, (2002); ***(7)*** Tuch DS et al, Proc. ISMRM, Denver, USA, p. 791, (2000); ***(8)*** Assaf Y et al, *Magn Reson Med* 47:115-126, (2002).