Limitations of Condition Number as an Optimization Metric for Diffusion Gradient Schemes

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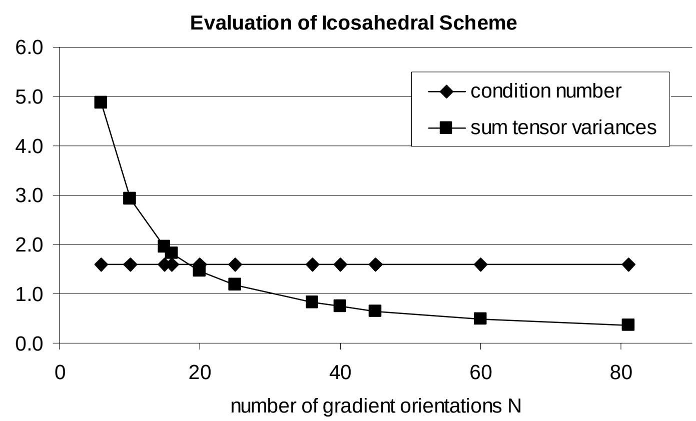
INTRODUCTION. The condition number has been used to evaluate DTI gradient schemes [1], but schemes obtained via minimization of that metric are not as rotationally invariant as the icosahedral set [2]. This suggests a limitation to condition number as an optimization metric. Partly to address this issue, an analytical formalism for characterizing the propagation of error throughout the entire DTI computational chain was recently introduced [3]. This formalism incorporates measurement noise (i.e., noise in the raw diffusion-weighted images) as well as gradient scheme orientations into the least-squares design matrix. The formalism yields an intrinsic metric, the sum of the dimensionless tensor variances $\Sigma\sigma_{bD}$, with which we have assessed a number of diffusion-weighting gradient schemes from the literature to investigate its potential merit for optimization of DTI acquisitions.

METHODS. The analytical formalism for DTI calculations was described in reference [3], with $\Sigma\sigma_{bD}$ defined as Equation (4). The $\Sigma\sigma_{bD}$ metric and the condition number were calculated for selected gradient schemes from the literature: icosahedral [2], electrostatic repulsion [4], and downhill-simplex minimization (DSM) of condition number [1]. Since condition number is insensitive to measurement noise, the measurement noise was assumed to be unity for purposes of calculating $\Sigma\sigma_{bD}$, ensuring a fair comparison.

RESULTS. The survey of condition number and $\Sigma\sigma_{bD}$ values is listed in Table 1. The condition number is not sensitive to increased number of gradient directions (N), but the $\Sigma\sigma_{bD}$ values decrease monotonically as N increases for all schemes listed in the table. The results of both metrics for the icosahedral scheme are plotted in Figure 1.

Table 1

	Icosahedral		Electrostatic		DSM	
N	cond	$\Sigma\sigma_{bD}$	cond	$\Sigma\sigma_{bD}$	cond	$\Sigma\sigma_{bD}$
6	1.581	4.875	1.583	4.875	1.323	4.863
10	1.581	2.925	1.624	2.934	1.324	2.926
12			1.587	2.438		
13			1.599	2.259		
15	1.581	1.950	1.615	1.966		
16	1.581	1.828	1.581	1.827		
20	1.581	1.463	1.615	1.471	1.324	1.463
21			1.600	1.391		
25	1.581	1.170	1.584	1.171		
27			1.585	1.084		
30			1.595	0.975	1.323	0.975
36	1.581	0.813				
40	1.581	0.731			1.323	0.731
45	1.581	0.650				
55			1.585	0.532		
60	1.581	0.488				
81	1.581	0.361				



DISCUSSION. The $\Sigma\sigma_{bD}$ metric describes the precision of the diffusion tensor estimate, which directly affects the quality of anisotropy calculations [3]. For the schemes we have investigated, the $\Sigma\sigma_{bD}$ metric demonstrates that there is a significant improvement in the precision of tensor calculations when increasing diffusion gradient sampling (i.e., the design matrix is over-determined) when the noise in the raw diffusion-weighted images is held constant. Therefore, relying on condition number alone to distinguish the merits of a given diffusion scheme over another may be insufficient.

With constant measurement noise, the $\Sigma\sigma_{bD}$ metric mathematically reduces to the κ index introduced by Papadakis et al [5], though the $\Sigma\sigma_{bD}$ metric has applications beyond κ due to its capability of accounting for noise . The results in Figure 1 suggest that using the minimum N = 6 with more signal averages (NEX) is not a guarantee of noise performance. Likewise, it can be shown that $\Sigma\sigma_{bD}$ increases as NEX decreases for a given N, since the metric accounts for noise dependence (unlike condition number or κ). Therefore, $\Sigma\sigma_{bD}$ may be useful in optimizing the inherent tradeoff between N and NEX, when designing DTI acquisition strategies.

CONCLUSION. The condition number may not be sensitive to the advantages of increased N, especially for the icosahedral family of diffusion gradient schemes. In contrast, the $\Sigma\sigma_{bD}$ metric does demonstrate a sensitivity to increased diffusion gradient sampling.

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