Spatial Analysis of structural MR data using a geostatistical method: Empirical Variogram Approach

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Introduction

One of the key difficulties in modeling the spatial dependency of the MR/fMRI signal intensity in the human brain is the inhomogeneity arising from the presence of three different tissue types (gray matter, white matter and CSF). This gives rise to discontinuity at the boundaries of the surfaces of these constituents of the brain. Here we adopt a segmentation based approach to isolate gray matter, white matter and CSF that makes each individual segment more homogeneous than the whole brain and a spatial modeling becomes more feasible. For the sake of simplicity, we will ignore the temporal evolution and consider only the spatial evolution as a stationary random process. To characterize the spatial continuity or roughness of MR data, an analysis based on the use of variograms is proposed, which is a technique widely used for modeling geostatistical processes. Under the assumption of stationarity, we can estimate parameters for the covariance model for each segment and further compare the consistency of the model and its parameters for normal healthy subjects as well as subjects diagnosed to have dyslexia. For computational efficiency, the analysis is performed on individual slices of the brain instead of using a 3D approach. With a sufficient number of subjects, it may be possible to construct predictive intervals for spatial covariance parameters. A subject with parameters falling outside of the predictive intervals may be further scrutinized for any potential structural anomalies. This approach allows us to divide the brain further into sub-areas and to identify more precisely the regions with unusual covariance parameters, which may have clinical significance. In a more general perspective, this analysis may be used for spatial interpolation using kriging and kernel density methods.

Theory and methods

Let $X(q(\boldsymbol{u}))$ be the observation at a voxel q with coordinates $\boldsymbol{u}$. For simplicity we can write $X(\boldsymbol{u})= X(q(\boldsymbol{u}))$. Then $X(\boldsymbol{u})$ may be considered as a spatial random process. We assume that $X(\boldsymbol{u})$ is a stationary process, i.e. the covariance $\mathrm{cov}(X(\mathbf{u}),X(\mathbf{v}))= \Omega(\|\mathbf{u}-\mathbf{v}\|)$ depends only on the distance between the two points **u** and **v**. Variogram analysis consists of an experimental semi-variogram *calculated from the data*, which is a non-parametric step, and also *fitting a model to the semi-variogram*, which is a parametric step. At first we calculate the semi-variogram defined by $V(\mathbf{u},\mathbf{v})=0.5E[X(\mathbf{u})-X(\mathbf{v})]^2$, which is related to the covariance function by $V(\mathbf{u},\mathbf{v})=\mathrm{cov}(0)-\mathrm{cov}(\|\mathbf{u}-\mathbf{v}\|)$. Although $\mathrm{cov}(\mathbf{0})$ must be the fixed variance, $\mathrm{cov}(\mathbf{0}+)$ may be significantly larger than cov(0). In geostatistics this is known as a nugget effect and could arise from a microscale variation in data. There exist a number of standard forms of covariance functions, which are commonly used. A nugget effect is usually added to each of them. In our method we considered the exponential covariance function $\Omega(\boldsymbol{u},\boldsymbol{v})=\rho+\sigma^2 e^{-d/r}$, where σ^2 is a sill, r is a range, ρ is a nugget and d is the distance between voxels with locations u and v. We have estimated sill σ^2, range r and nugget ρ by fitting the exponential covariance function $\Omega(\boldsymbol{u},\boldsymbol{v})$ to the calculated semi-variogram by the weighted least squares method (WLS) with weights being proportional to the number of pairs per variogram bin as well as using non-linear least squares (NLS) method. Both methods provide relatively the same parameter estimates of covariance model while NLS has been more computationally efficient. The obtained covariance parameters have been compared for each constituent of the brain and separately for the normal and dyslexic subjects. The approximated density functions have been plotted for each of the parameters using the estimated values for different slices of the brain. As a further extension of this approach one can consider a general Matern family of covariance functions which includes the most used exponential and Gaussian covariance forms. However, our numerical results show that even a simple exponential covariance function is relatively stable and consistent for both test groups and each constituent of brain. The estimation process can be further improved using non-Euclidian metrics, as the conventional Euclidean distance function may not be appropriate for segments with holes, as in gray matter.

Results

We apply our analysis to 4 normal subjects and 4 subjects with dyslexia, who were scanned with a GE 1.5T scanner using a standard 3D SPGR pulse sequence. In Figures 1 and 2, we provide the estimated density functions of the sill σ^2, the range r and the nugget ρ for white matter (WM) and gray matter (GM) for normal subjects and subjects with dyslexia, respectively. The normal subjects show tendencies to bimodal behavior of range parameter in WM and more significant small-scale variability (nugget variability) for GM. Inter-subject variability for sill also appears to be less for normal subjects. However, such patterns may be due to outlier influence or lack of data. Although the results are preliminary, the plots show the relative consistency of fitting the exponential covariance function to the spatial data, and we found this approach to be promising. To test our method further, we plan to apply the method to a higher number of normal and affected subjects.

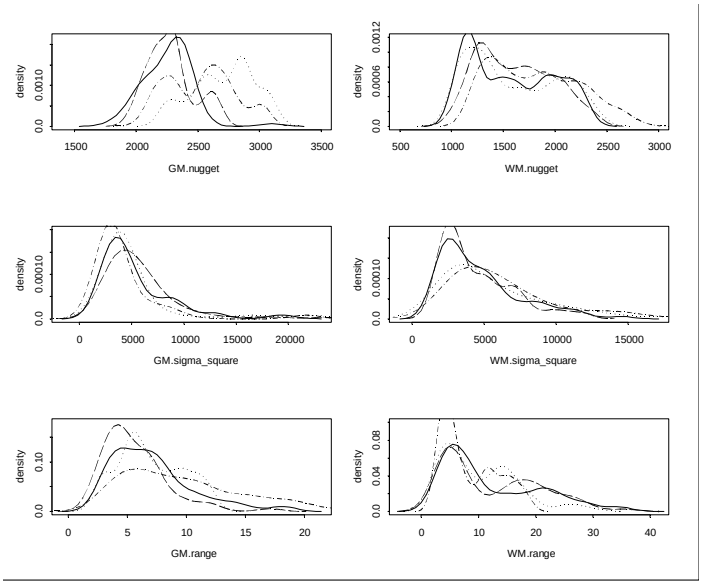

Figure 1. The density functions of covariance parameters for normal subjects.

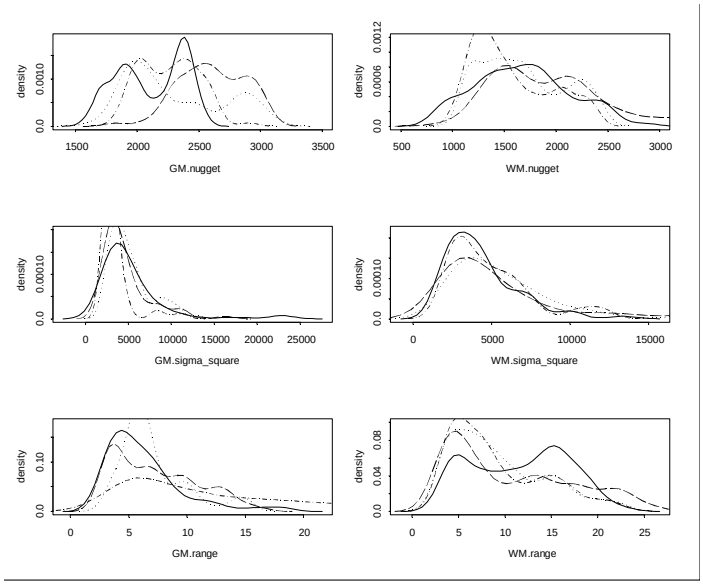

Figure 2. The density functions of covariance parameters for dyslexic subjects.

References

H. Wackernagel (1998), "Multivariate Geostatistics", Springer-Verlag.