Inter-scan motion correction in fMRI. Direct k-space determination of the 3D rotation parameters

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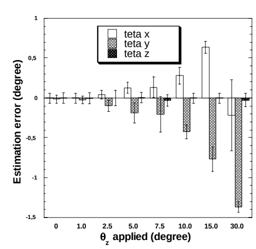
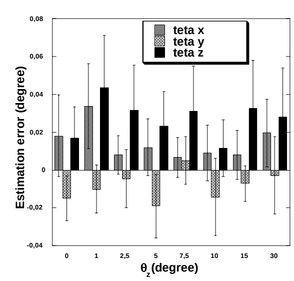
Introduction. Fourier registration algorithms have been shown to enable direct estimation of the motion parameters in ***k***-space [1-2]. The computational efficiency of these methods relies upon the possibility of decoupling the estimations of the rotation and translation parameters. Two fMRI Fourier registration algorithms have been described [3-4]. One of these [3] is limited to in-plane displacements and has been shown to be very efficient and accurate. With the second algorithm, applicable to the 3D case, determination of the rotation axis has proven difficult. Here, we have developed an alternative, non-iterative, 3D registration technique based on Fourier analysis. The method reduces the determination of the 3D rotation parameters to that of three in-plane rotations, assuming small rotation angles.

Theory. Let us consider the noiseless ***k***-space signals $s(\mathbf{k})$ and $s_{RT}(\mathbf{k})$ from two 3D objects $S(\mathbf{r})$ and $S_{RT}(\mathbf{r})$ which merely differ by their position in ***r***-space. The corresponding motion may be described in terms of translation and rotation operators T and R : $S_{RT}(\mathbf{r}) = RT\, S(\mathbf{r})$. R thereby describes a rotation of angle $\theta_{\mathbf{u}}^{R}$ about a rotation axis ***u*** through the origin. The autocorrelation functions (ACs) $F(\mathbf{r})$ and $F_{RT}(\mathbf{r})$ of the objects $S(\mathbf{r})$ and $S_{RT}(\mathbf{r})$ are insensitive to translations. We therefore determine the rotation parameters from the ACs rather than from the objects. Assuming small rotations, R may be decomposed into a *commutative* product of three rotations R_x, R_y, R_z about the *x*, *y* and *z* directions and of angles θ_x^R, θ_y^R, θ_z^R, respectively. The problem of determining ***u*** and $\theta_{\mathbf{u}}^{R}$ may thus be reduced to that of determining the three (small) rotation angles about the *x, y,* and *z*-axes. Assume now that we are interested in determining one of those angles, say θ_z^R. Given the small angle hypothesis, the slices of the ACs positioned at *z* =0 are simply related : $F_{RT}(\rho,\theta_z) \approx F(\rho,\theta_z - \theta_z^R)$, where ρ and θ_z are the polar coordinates in the *z*=0 plane. If $f(\rho,\xi_z)$ and $f_{RT}(\rho,\xi_z)$ are the *1D* Fourier transforms of $F(\rho,\theta_z)$ and $F_{RT}(\rho,\theta_z)$, respectively, then one easily verifies that $f(\rho,\xi_z)\, f_{RT}^{*}(\rho,\xi_z) = \left[\exp i\varphi(\xi_z)\right]\left|f(\rho,\xi_z)\right|^2$, with $\varphi(\xi_z) = 2\pi\theta_z^R\xi_z$, permitting determination of θ_z^R from the phase $\varphi(\xi_z)$. The search for θ_z^R may be performed on the *2D* Fourier transforms $g^z(k_x,k_y)$ and $g_{RT}^z(k_x,k_y)$ of $F(x,y,0)$ and $F_{RT}(x,y,0)$ as well. These Fourier transforms are the projections P_{k_z} along k_z of the power spectra of the objects. We determine θ_z^R from these projections. In the presence of noise, the rotation angle θ_z^R may then be estimated by least square fitting the model function $\phi(\xi_z) = 2\pi\theta_z\xi_z$ to the unwrapped phase $\varphi(k_{xy},\xi_z)$ of $h^z(k_{xy},\xi_z)h_{RT}^{z*}(k_{xy},\xi_z)$, where $h^z(k_{xy},\xi_z)$ and $h_{RT}^z(k_{xy},\xi_z)$ are the FTs of $g^z(k_{xy},\theta_z)$ and $g_{RT}^z(k_{xy},\theta_z)$, respectively. The rotation angles θ_x^R and θ_y^R are estimated in the same manner as the rotation angle θ_z^R. Once the three rotation angles have been determined, the three rotations $R_x(-\theta_x^R)$, $R_y(-\theta_y^R)$, $R_z(-\theta_z^R)$ may be applied successively to $S_{RT}(\mathbf{r})$ so as to obtain a translated-only image $S_T(\mathbf{r})$. At this point, it is important to keep in mind that the method relies upon the assumption that the rotations about the *x*, *y*, and *z*-axes commute and thus the order whereby these rotations are applied is irrelevant. If this assumption is expected to be invalid, a few *ad hoc* iterations of the procedure are recommended so that, eventually, the estimation may be performed under the hypothesis of small angles. Following correction of the rotations, the translation can be corrected for with the procedure described by Maas et al [3].

Material and methods. The method was assessed by means of a set of Monte-Carlo experiments. We thereby used a simulated, noiseless, T1-W brain image from the MNI database. Various 3D rotations were applied. Different levels of SNR of the image were considered, within the range 10-100.





Results and Discussion. As an illustration, Fig. 1 (left) shows the registration errors on the direct (single pass) estimates of θ_x^R, θ_y^R, θ_z^R, for an SNR of 50 and a rotation applied about the z-axis. The registration errors on the estimates of θ_z^R are very small for all rotation angles θ_z^R applied (up to 30°). For large rotation angles, the accuracy of the estimates of θ_x^R and θ_y^R improves significantly if the process is iterated even a single time (Fig.2), following correction for the largest rotation angle estimate.

References. 1. Cideciyan AV et al, SPIE Med Imag VI, 310 (1992). 2. De Castro et al, IEEE Tr.Patt.Anal.Mach.Intell. 9: 700 (1987). 3. Maas L. et al. MRM 37: 131 (1997). 4. Kassam A. et al, JMRI 6: 620 (1996)