

# Measurement of thermal diffusivity by magnetic resonance imaging

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## Introduction:

MR thermometry has potential use for monitoring temperature changes during thermotherapy. A method for monitoring the transient diffusion of thermal energy during the procedure could improve the precision and the effectiveness of treatments. MR methods developed in the past have used relaxation times, diffusion coefficient of water, proton resonance frequency and phase shift to monitor temperature, but the explicit measurement of the thermal diffusivity of materials by MRI has not been reported before. A method based on measuring the equilibrium magnetization has been developed for measuring transient thermal diffusion effects with potential applications in MR thermotherapy and materials science. In this method a thermal pulse is applied to a medium and the resultant temporal variation of nuclear magnetization produced by the thermal pulse is monitored at a spatial distance. The resultant temporal variation of magnetization is a characteristic of the substance and from this the thermal diffusivity of the substance can be determined experimentally.

## Theory:

For a gradient echo imaging sequence with flip angle  $\theta$  [1]

$$M(t, T) = M(0, T) \sin \theta e^{-\frac{T_p}{T_1(T)}} \left( 1 - e^{-\frac{T_p}{T_1(T)}} \right) \left( 1 - e^{-\frac{T_p}{T_1(T)} \cos \theta} \right)^{-1} \quad (1)$$

The equilibrium magnetization,  $M(0, T) = N\mu^2 B_0 / k_B T$  for a spin of  $I=1/2$  [2], exhibits a negative temperature dependence for substances at temperatures commonly encountered in physiology and thermotherapy.

Then,

$$\frac{\partial \ln M(t, T)}{\partial T} = -\frac{1}{T} + \frac{T_E}{T_1^2(T)} \frac{\partial T_2(T)}{\partial T} + \cot \theta \frac{\partial \theta}{\partial T} - \frac{T_E}{T_1^2(T)} \frac{\partial T_1(T)}{\partial T} \frac{E_1}{1-E_1} - \left[ \sin \theta \frac{\partial \theta}{\partial T} - \cos \theta \frac{T_E}{T_1^2(T)} \frac{\partial T_1(T)}{\partial T} \right] \frac{E_1}{1-E_1 \cos \theta} \quad (2)$$

indicates that the temperature dependence of magnetization is a function of temperature, echo time, recovery time, spin-spin and spin-lattice relaxation times, and the dependence of relaxation times on temperature.

We consider the following cases

$$T_E \gg T_1(T), \quad \left| \frac{\partial \theta}{\partial T} \right| \ll 1 \quad (3)$$

Then, Eq. (2) simplifies to

$$\frac{\partial \ln M(t, T)}{\partial T} = -\frac{1}{T} + \frac{T_E}{T_1^2(T)} \frac{\partial T_2(T)}{\partial T} \quad (4)$$

which can be used to monitor the temporal variation of thermally induced magnetization.

From Einstein's diffusion theory [3]

$$\frac{1}{2} (M_1 - M_2) \Delta \Delta = -\alpha \frac{dM}{dz} A \tau \quad (5)$$

where  $A$  is the cross sectional area,  $\Delta$  is the diffusion length and  $\tau$  is the diffusion time. Defining the magnetization gradient over  $\Delta$  as

$$-\frac{dM}{dz} = \frac{M_1 - M_2}{\Delta} \quad (6)$$

the thermal diffusion coefficient becomes

$$\alpha = \frac{\Delta^2}{2\tau} \quad (7)$$

At a spatial distance,  $\Delta$ , from the thermal source, the temporal variation of magnetization will reach a maximum at a time,  $\tau$ , from which the thermal diffusivity can be calculated using the Eq. (7).

## Experimental Methods:

The feasibility of measuring  $\alpha$  has been demonstrated with MR imaging. A sample of glycerol in a cylindrical cell (10 mm dia., 50 mm long) with its axis aligned with the direction of  $B_0$  field was subjected to a thermal pulse at  $\Delta=0$  by a laser. The subsequent temporal variation of magnetization was monitored in a coronal plane by GRE imaging using a 2 Tesla Bruker NMR System. The imaging parameters were; BW 16 kHz,  $T_E = 8$  ms,  $T_R = 90$  ms, FOV=6.4 cm, 128x128, slice thickness = 4 mm, flip angle =  $\pi/4$ , and NEX = 60. The signals from voxels along the read direction were averaged and plotted vs time. The direction of heat diffusion was taken to be along the direction of  $B_0$  field.

## Results and Discussion

The temporal variation of magnetization, following a thermal pulse applied at  $t=0$  s is given in Fig. 1. The  $\partial T_2(T) / \partial T = 0.0013$  s/K as measured near room temperature indicates that the temperature dependence of magnetization at a echo time of 8 ms is positive for glycerol as shown in Fig. 1. The experimental data given in Fig. 1 are similar to the simulated data for glycerol as shown in Fig. 2. A plot of  $\Delta^2$  vs.  $\tau$  gives a curve with a slope equal to  $2\alpha$  for the substance. Using the experimental data in Fig. 1, the thermal diffusivity of glycerol was calculated as  $0.94 \cdot 10^{-7} \text{ m}^2/\text{s}$  which is close to the previously known value of  $0.95 \cdot 10^{-7} \text{ m}^2/\text{s}$ .

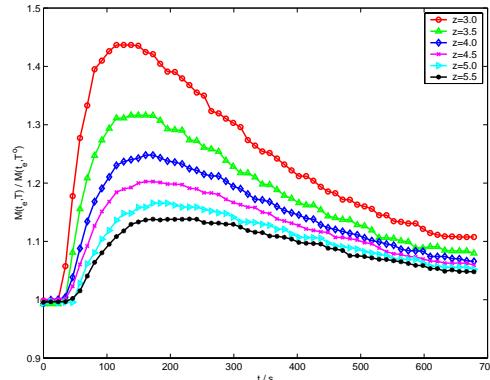


Figure 1: Experimental temporal variation of normalized magnetization vs. distance for glycerol in a 2 Tesla magnet following a laser pulse. Each curve corresponds to a distance  $\Delta$  in millimeters.

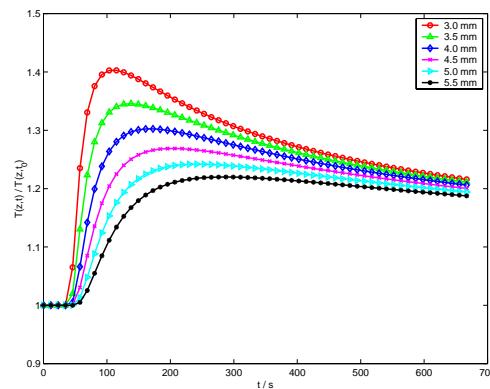


Figure 2: Simulated temporal variation of normalized temperature vs. distance for glycerol following a thermal pulse.

MR imaging can be used to monitor the transient diffusion of thermal energy during thermal application. MR methods can be used to measure thermal diffusivity of materials and tissues which is an important factor in thermotherapy.

## References

- [1] F. Bloch. Phys. Rev. 70 (7-8) (1946).
- [2] P. Langevin. CNRS, Paris. (1950).
- [3] A. Einstein. Ann. Phys. (1905).