Hemispherical gradient coils for MRI

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Introduction

Spherical gradient coils offer an interesting prospect for MRI because the three orthogonal field gradients may each be generated by current distributions composed of single spherical harmonics. Wires that approximate such harmonics may be wound on the surface of a sphere, giving a very pure gradient field inside and a maximally efficient coil [1]. Unfortunately the wires must be placed over the whole spherical surface to ensure that the harmonics are correctly represented, making access to the interior imaging volume problematic. A potential approach to overcoming this problem is to truncate the spherical coil to a hemisphere, leaving half of the wires in place. Although access to the imaging volume is thus greatly improved, such a coil will not provide a useable field variation as the spherical harmonic is no longer well represented. However, as the present work demonstrates, it is possible to introduce additional truncated harmonics into the stream function so as to ameliorate the effect of truncation on the field. The resulting hemispherical gradient coils have a geometry that is ideally suited for use in insert head gradient coil sets for clinical imaging, yielding a performance that is significantly better than their cylindrical counterparts.

Theory

A general current distribution on the surface of a sphere can be represented by a stream function that is composed of a weighted sum of spherical harmonics. We have derived expressions for the magnetic field produced by such a stream function and for the stored energy, which can be related to the inductance of a final coil design. This has allowed evaluation of the ultimate achievable performance of axial and transverse spherical coils for the first time. In the case of a hemispherical gradient coil, the stream function is initially expressed as a weighted sum of spherical harmonics truncated to half the surface of the sphere. The appropriate forms of the stream function in the region where $0 < \theta < \pi/2$, for *z*- and *x*-gradients are shown below, where c_l is the weighting while $P_l(\cos\theta)$ and $P_{lm}(\cos\theta)$ represent the Legendre and associated Legendre functions. As in the case of cylindrical coils, a *y*-gradient may be generated via rotation of the *x*-gradient stream function by 90° about the *z*-axis.

$$z\text{-gradient: } \psi(\theta,\phi) = \sum_{l=1}^{N} c_l [P_l(\cos\theta) - P_l(0)P_0(\cos\theta)]\,; \qquad x\text{-gradient: } \psi(\theta,\phi) = \sum_{l=1}^{N} c_l [P_{l,1}(\cos\theta) - P_{l,1}(0)P_{1,1}(\cos\theta)]\cos\phi$$

Using the properties of spherical harmonics it is possible to represent each truncated spherical harmonic as an infinite sum of spherical harmonics on the full sphere. The expressions derived for full-sphere harmonics may then be used to calculate the magnetic field generated by a hemispherical stream function, along with the stored energy. To design a hemispherical coil, a least-squares minimization of the deviation of the field from a pure gradient at a set of constraint points is performed, yielding the appropriate values for the weightings of the spherical harmonics, c_l , in the hemispherical stream function. The inductance can also be included in the minimization if desired. Its contribution relative to that of the field optimization can be adjusted to produce a coil which has the lowest possible inductance whilst generating an adequately homogeneous gradient over the region defined by the constraint points. Contours of the stream function yield wire paths used for coil fabrication and field validation with the Biot-Savart law. Although there is no net force or torque on an axial hemispherical coil carrying current in the presence of the static magnetic field and the net force on a transverse coil is also zero, transverse coils are not intrinsically torque-balanced. Torque-balancing of transverse coils may however be achieved by including an additional truncated harmonic in the stream function whose weighting is chosen to cancel out the torque contributions from all other harmonics.

Methods

Full-sphere coils of arbitrary size were designed using stream functions composed of appropriate pure spherical harmonics. A comparison of the field calculated analytically from truncated spherical harmonics and that found from applying the Biot-Savart law to the contours of the hemispherical stream function was made. Optimised, minimum inductance, axial and transverse hemispherical gradient coils were designed specifically for brain imaging. The hemisphere diameter was chosen to be 35 cm, with an 11 cm diameter spherical homogeneous region, whose centre was located 7.5 cm from the edge of the hemisphere. Field constraint points were evenly spaced throughout the spherical region of interest.

Table 1	*z*-gradient coil	*x*-gradient coil (unbalanced)	*x*-gradient coil (balanced)
$\eta^2 a^5/L$ ($T^2m^3A^{-2}H^{-1}$)	3.29×10^{-7}	2.43×10^{-7}	7.21×10^{-8}
# of discretization levels	20	20	20
Efficiency, η ($Tm^{-1}A^{-1}$)	4.52×10^{-4}	3.29×10^{-4}	1.82×10^{-4}
Inductance, *L* (μH)	102	72.0	75.8
Diameter × Length of Cylindrical hom. vol.	12 cm × 15 cm	16 cm × 14 cm	8 cm × 7 cm

Results

The spherical coils displayed almost perfect gradient linearity within the interior of the sphere. The number of wires used in the discretization process, and thus the accuracy with which the harmonics were represented, determined the extent of the region of linearity. For example with 10 wires per symmetry unit, the spherical volume within which the gradient deviates from linearity by less than 5% has a radius of 0.9*a* and 0.71*a* for *z*- and *y*-coils respectively, where *a* is the coil radius. The figures of merit formed from the ratio of efficiency squared to inductance for the axial and transverse coils are $6 \times 10^{-7} a^{-5}$ $T^2m^{-2}A^{-2}H^{-1}$ and $4.5 \times 10^{-7} a^{-5}$ $T^2m^{-2}A^{-2}H^{-1}$ respectively. The figure for the axial coil is marginally higher than that of a minimum inductance spherical coil previously reported [1]. Using 50 full sphere harmonics to represent each truncated hemispherical harmonic was found to yield magnetic field values which are in excellent agreement with those calculated numerically from discrete wire paths, thus verifying the mathematical formalism employed here. Table 1 summarises the properties of the hemispherical head gradient coils that have been designed. The wire paths and field contours for the hemispherical *z*-gradient coil and the unbalanced hemispherical *x*-gradient coil are shown in Fig. 1. As expected, the performance is not as good as the full sphere coils, but the figures of merit compare very favorably with those of cylindrical head gradient coils of similar diameter and homogeneous volume. For example, an optimized axial head gradient coil of 37 cm diameter with a slightly larger homogenous volume has a value of $\eta^2 a^5/L$ of 1.03×10^{-7} $T^2m^3A^{-2}H^{-1}$ [2], while an asymmetric transverse coil of 32 cm diameter with a somewhat smaller homogeneous volume has $\eta^2 a^5/L = 5 \times 10^{-8}$ $T^2m^3A^{-2}H^{-1}$ [3]. A small (a = 6.4 cm), prototype hemispherical *z*-gradient coil was fabricated and its efficiency and inductance measured. The resulting values were in excellent agreement with calculations. The torque-balanced coil has a smaller region of linearity than the unbalanced transverse coil, the extra retrograde current paths having a significant effect on the field profiles. Alternative approaches to torque balancing are now being explored.

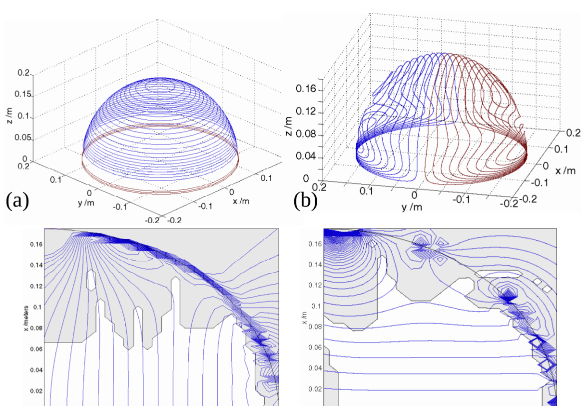

Figure 1: (a) hemispherical *z*-gradient; (b) hemispherical *x*-gradient; red denotes reversed current paths; (c) field contours for *z*-gradient; (d) field contours for *x*-gradient; the homogeneous region (less than 5 % deviation from linearity) is unshaded.

References

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