NURBS in DT-MRI

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Introduction:

Diffusion Tensor MRI (DTI) measures a diffusion tensor of water in tissue, from which useful microstructural information can be obtained [1]. DTI provides information about the local diffusion field from which one can attempt to reconstruct nerve fiber tracts (e.g., see [2]). One problem in white matter tractography has been that the diffusion tensor data from which these continuous tracts are constructed is actually discrete and noisy [3]. Several approaches have been proposed to provide a continuous tensor field approximation to the discrete, noisy tensor data so that the fiber tract trajectory can be followed continuously through the imaging volume. One such approach employs B-splines [4]. The problem with this approach is that it removes noise by sacrificing resolution. Also this method fails to provide reliable estimates of useful quantities that arise in differential geometry, like curvature and torsion, which provide descriptions of geometric features of the tract, because these quantities involve taking high-order derivatives of functions of the tensor field. Increasingly, we see differential geometric features as providing new and useful parameters to help us characterize different white matter pathways. We argue that a class of basis functions, known as Non-Uniform Rational B-Splines (NURBS), provide a suitable tool to achieve such goal.

Theory and Methods:

NURBS are a generalization of the B-splines (as well as rational Bezier curves and surfaces). Their numerical evaluation can be implemented efficiently and stably using relatively fast algorithms. They are recursively related to the splines of lower order/degree, and the derivatives can be evaluated easily in recursive fashion too. The curves represented with NURBS are also invariant under affine as well as perspective transformations. But, in addition to these features it shares with ordinary B-spline, the NURBS can very efficiently represent, not only piecewise polynomial curves and surfaces, but also conic sections (circles, ellipses, hyperbolas, parabolas) and, most importantly, they efficiently represent the free-form curves with arbitrary shapes. NURBS provide a parametric description of curves and surfaces using *control points*. These control points define a *control polygon*. With each control point there is associated a single weight parameter that controls the shape of the curve. The higher the weight the more the curve is drawn towards the control polygon (figure 1). Thus, in N_d-dimensional space the total number of parameters is N_p=N_c (N_d+1), where N_c is the number of the control points. This same relatively simple model can represent complicated shapes as well as many simple geometric objects. For example, they can represent a circular arc "exactly" with only 3 control points, while regular B-splines would require infinitely many parameters to do that. A potential additional parameter called the *knot vector* which controls how the non-uniform segments subdivide the domain of the curve. Usually, knot vector is assigned based on the placement of the control points. A number of efficient algorithms for fitting data to NURBS models have already been developed [5]. Here we show some of their applications to DT-MRI, and discuss their potential.

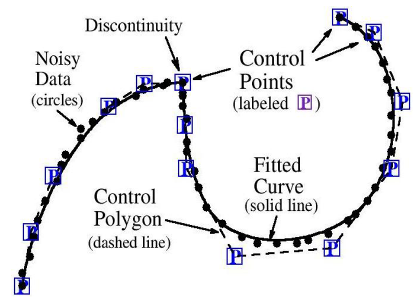

Figure 1. NURBS fit and parameters

Results and Discussion:

Figure 1 shows a fit to artificial data (noisy samples of two different curves were joined together to create an apparent discontinuity. The solid line indicates the fit obtained using algorithm A9.10 in [5], in which an error bound is specified. The fit returns the control points and weight parameters and one can see that even with discontinuity, the NURBS describes the curve very well with only 14 control points. It achieves this by placing a control point where the discontinuity occurs and assigns it a very large weight parameter. Figure 2a shows a 2D-projection of a tract (yellow line) onto a slice of DT-MRI volume with color coded orientations. The tract passes through corpus callosum and spans from one end of the brain to the other. Figure 2b shows that same tract in 3D, parameterized with B-spline coefficients (3*130=390 parameters) on the right and NURBS (N_c=15, N_p=60) on the left. Such sparse parameter space enables more efficient explorations of connectivity. Finally, we address the issue of estimating curvature, which is problematic for noisy data. Figure 3a shows an artificial pathway consisting of 4 adjoined circles with radii 100, 10, 5, and 35 in arbitrary units. In figure 3b the estimates of the radius of curvature are compared with the actual curvatures (solid blue line). NURBS estimates (solid black line) outperform B-spline continuous approximation [4] for any level of approximation, interpolation i.e., Δ=1, (triangles), Δ=0.5, i.e. twice smoother, (dots with dashed line) and Δ=0.2 (solid green line). Results are plotted on a logarithmic scale due to the significant range in the estimates. Note that the spike in the NURBS estimate of the radius is occurring at the inflection point where the curvature is zero. We conclude that NURBS are well suited for exploring differential geometry in DT-MRI.

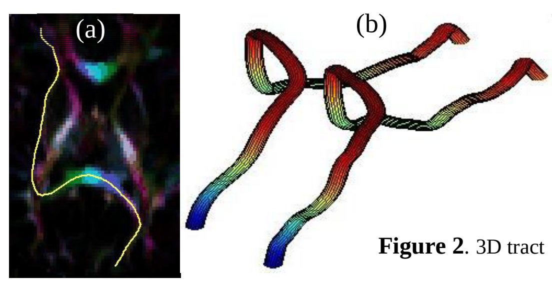

Figure 2. 3D tract

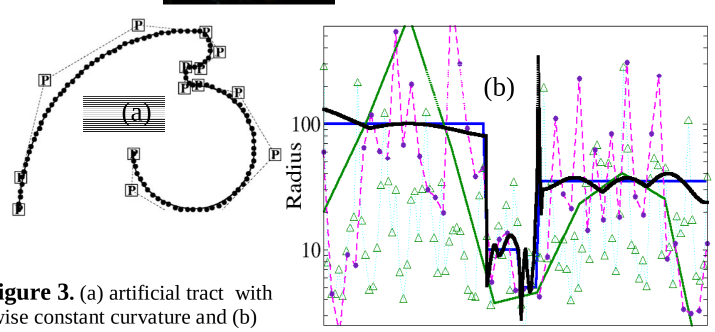

Figure 3. (a) artificial tract with piecewise constant curvature and (b) NURBS and B-spline estimates

References:

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