Voxel Power for Repeated Measures ANOVA with Nonsphericity

M. McAvoy[1], T. Brown[1], H. Burton[1], B. Schlaggar[1], S. Petersen[1]
[1]Washington University, St. Louis, MO, United States

Introduction

Statistical power has been addressed for fixed effects with PET data [1]. We wish to address voxel level power for a repeated measures analysis of variance (ANOVA) with fMRI data. The analysis is a mixed effects analysis with subjects as the random factor and the experimental conditions as fixed effects. When the analysis includes repeated factors at more than two levels, the nonsphericity of the underlying covariance matrix must be accounted for when calculating power.

Theory

Power for repeated measures ANOVA with nonsphericity [2,3] can be calculated with the incomplete beta function [4]

$$Power = I_{\frac{df_2}{df_2 + df_1 F_\beta}}\left[\frac{df_2}{2}, \frac{df_1}{2}\right], \text{ with}$$

$$F_\beta(df_1, df2) = \frac{\varepsilon \nu_1}{\varepsilon \nu_1 + \lambda} F_{crit}\left(1-\alpha, E(\tilde{\varepsilon})\nu_1, E(\tilde{\varepsilon})\nu_2\right).$$

F_β is the F value for the probability of making a type II error (i.e. failing to reject the null hypothesis) with degrees of freedom $df_1 = \frac{(\varepsilon \nu_1 + \lambda)^2}{\varepsilon \nu_1 + 2\lambda}$ and $df_2 = \varepsilon \nu_2$ and noncentrality parameter $\lambda = \varepsilon \nu_1 F_A$. F_{crit} is the critical value for the F test. F_A is the F value observed under the alternate hypothesis. The correction for nonsphericity ε, $0 < \varepsilon \le 1$, is calculated from the covariance matrix of the repeated measures ANOVA [5]. The expected value of the random variable $E(\tilde{\varepsilon})$ is the population estimate of $\tilde{\varepsilon}$, an observed random variable.

Methods

Ninety five subjects aged 7 to 32 years (mean 15.3 years, 52 female) participated in a fMRI language experiment similar to that described in [6]. In response to seen and heard words, the subject had to either overtly generate a verb, name a word of opposite meaning, or provide a word that rhymed. The indvidual subject's BOLD data were fit with a general linear model [7] (GLM) using a delta function basis set. The GLM estimates were input in a repeated measures ANOVA that included a single factor of time at seven TRs (TR=3.08s) representing the combined BOLD response to correctly answered items across the verb generation, opposite, and rhyming tasks. This simple main effect of time map identifies regions of the brain that have a hemodynamic response and was used to provide estimates of F_A. The nonsphericity correction due to the stimulus paradigm for this analysis had a value of 0.82, which was used for both ε and $E(\tilde{\varepsilon})$ in the power analysis.

Results

Figure 1 shows the power to detect a hemodynamic response at α=1e-28 (i.e. z~11 [6]) for samples of 8, 12, and 16 subjects. Power ranges from 0.01 to 1 (i.e. 1% to 100%).

Conclusions

The increase in power is quite dramatic from 8 to 16 subjects, with the latter showing very robust detection of hemodynamic responses throughout cortex. The value of alpha of may seem unreasonably large, but one must consider that this is a very robust effect across two modalities (visual and auditory) and three tasks (verb generation, opposites, and rhyming). This analysis made no assumptions as to the shape of the hemodynamic response function with the GLM using a delta function basis and the resulting estimates analyzed directly as factor levels in the repeated measures ANOVA.

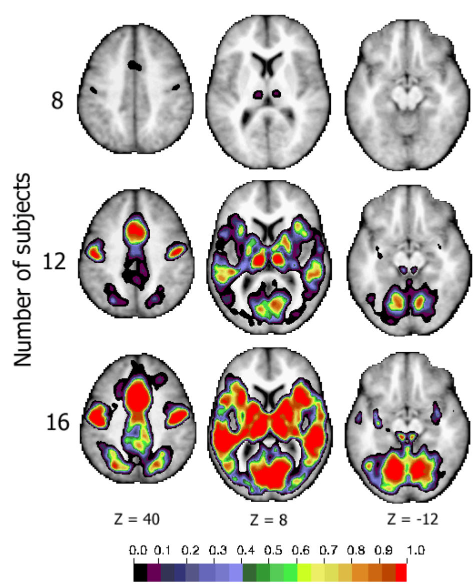

Fig. 1. Power to detect a hemodynamic response (α=1e-28, z~11) for samples of 8, 12 and 16 subjects.

References

1. Van Horn et al. NeuroImage 7, 97-107 (1998).
2. Muller and Barton. J American Stat Assoc 84, 549-555 (1989).
3. Muller et al. J American Stat Assoc 87, 1209-1226 (1992).
4. Patnaik. Biometrida 36, 202-232 (1949).
5. McAvoy et al. HBM submitted.
6. Schlaggar et al. Science 296, 1476-1479 (2002).
7. Friston et al. HBM 2, 189-210 (1995).