A Distributed Equivalent-magnetic Current Based FDTD Method for the Evaluation of the Eddy Currents Induced by Pulsed Gradient Fields

F. Liu[1], S. Crozier[1], Q. Wei[1]

[1]School of Information Technology and Electrical Engineering, Brisbane, Queensland, Australia

Synopsis

The finite difference time domain (FDTD) method [1] has been employed to calculate the eddy currents induced inside the human body by gradients [2-5]. Many technical problems have already been solved to some extent; however, the FDTD method has to be improved/modified further to effectively handle low-frequency gradient field modelling problems. To circumvent the mapping of the complicated gradient coil geometry into Yee's staggered meshes, a distributed equivalent magnetic current based FDTD method has been proposed. It has been demonstrated that how the total-field/scattered-field concept can be used to formulate a novel low-frequency FDTD algorithm, where the electromagnetic source can be simply obtained by quasistatic calculation of the empty coil's vector potential or measurements therein.

Introduction

In the current-source based FDTD algorithms [4,5], gradient coils are modelling as standard current/voltage source and mapped into staggered yee's cells; however, when the coils' geometry becomes too complex, some special source models are required. Here, a novel distributed equivalent magnetic currents (DEMC) based FDTD method is proposed to enable complex low frequency source modelling.

Theory

Based on the linearity of Maxwell's equations, the electric and magnetic fields can be expressed as the summation of the incident components and scattered components, *i.e.*, $E=E_i+E_s$, $H=H_i+H_s$. Where E_i,H_i are the values of the incident wave fields, which are easily obtained at all space points of the FDTD grid. E_s,H_s are the unknowns of the scattered wave fields, which are the results from the interaction of the gradient fields with the human body. If Faraday's law is written as $\nabla\times E=-\mu\partial H_i/\partial t-\mu\partial H_s/\partial t$, then $\mu\partial H_s/\partial t=-\nabla\times E-\mu\partial H_i/\partial t=-\nabla\times E+M_i$ where $M_i=-\mu\,\partial H_i/\partial t$ is the distributed magnetic current. Similarly, the Ampere's law can be written as $\nabla\times H_i+\nabla\times H_s=\sigma E+\varepsilon_0\varepsilon_r\partial E/\partial t$, it is noticed that $\nabla\times H_i=\varepsilon_0\partial E_i/\partial t$, so, $\varepsilon_0\partial E_i/\partial t+\nabla\times H_s=\sigma E_s+\sigma E_i+\varepsilon_0\varepsilon_r\partial E/\partial t$.

At low frequencies, $\sigma E_i ? \varepsilon_0\partial E_i/\partial t$, hence, $\nabla\times H_s=\sigma E_s+\sigma E_i-\varepsilon_0\frac{\partial E_i}{\partial t}+\varepsilon_0\varepsilon_r\frac{\partial E}{\partial t}\approx\sigma E_s+\sigma E_i+\varepsilon_0\varepsilon_r\frac{\partial E}{\partial t}=\sigma E+\varepsilon_0\varepsilon_r\frac{\partial E}{\partial t}$

So, the final formulations are:

$$\begin{cases}\mu\partial H_s/\partial t=-\nabla\times E-M_i\\ \varepsilon_0\varepsilon_r\partial E/\partial t=\nabla\times H_s-\sigma E\end{cases}\text{ Inside the body;}\qquad\begin{cases}\mu\partial H_s/\partial t=-\nabla\times E_s\\ \varepsilon_0\partial E_s/\partial t=\nabla\times H_s\end{cases}\text{ Outside the body}$$

In this way, the scattered component is only considered for H-field in all the space; for the E-field, the total field is considered inside the body and the scattered field is calculated in free space. The interaction with the gradient coil structure is ignored since it is minimal for gradient fields. The magnetic currents can be calculated easily using low-frequency electromagnetic formulations [1].

Simulations

The algorithm was implemented, with a 3D 12-layered PML [1]. The technique was compared with an approximate analytical solution [1] for a multi-layered conducting sphere. The gradient field data was obtained from a 20-loop z-gradient designed using a simulated annealing optimization method [6]. Fig.1 shows the calculated eddy currents inside the sphere.

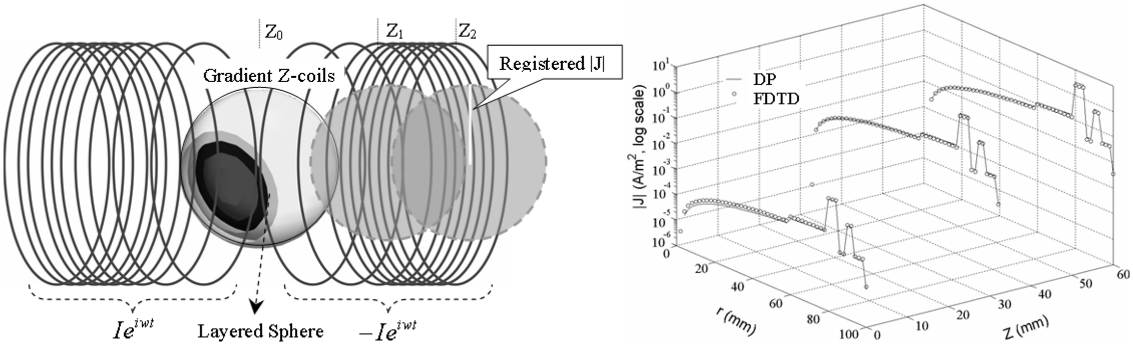

Fig.1. Calculation of the eddy currents inside the seven-layered spherical phantom excited by a typical Z- gradient coils [6]. (a,left) Schematic arrangement of the phantom inside the coils. To generate a slew rate of $dB/dz/dt\approx 100T/m/s$, the coil current is about 500.0A and the frequency is 1 KHz. The induced fields were calculated as a function of the phantom positions with respect to the gradients system. (b, right) Comparisons of Debye potential (DP) based analytic solution [1] and FDTD solution along the radial distance at y=0. The field values are shown for three different positions: z=0, 0.3, 0.6 m respectively.

Conclusion

The main advantage of this algorithm is does not need to map a complicated wire structure onto a staggered FDTD mesh. Furthermore, the computational resources required are dramatically reduced since the method only considers the space occupied by the human body.

References

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