A FDTD Model for the Calculation of Gradient-induced Eddy Currents in MRI Magnets

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Synopsis

This paper presents a 3-D FDTD scheme in cylindrical coordinates for eddy current calculations in magnet conductors. The singularity apparent in the governing equations is removed by using a series expansion method and the conductor-air boundary condition is encorporated using a variant of the surface impedance concept. The numerical difficulty due to the 'asymmetry' of Maxwell equations for low frequency problems is circumvented by taking advantage of the known penetration behavior of the eddy-current fields. A PML absorbing boundary condition in 3D cylindrical coordinates is also incorporated. The proposed numerical scheme is implemented, verified against a problem with a known analytical solution and an MRI-based numerical example is provided to illustrate the utility of the algorithm.

Introduction

When gradient coils are pulsed they induce eddy currents in the cryostat walls and other metallic parts of the MRI magnet. Distortions due to eddy currents are a major cause of concern in MRI because they degrade resolution, cause mis-registration and intensity/phase variations in both images and spectra. It is useful to electromagnetically analyze the behavior of the eddy currents to investigate methods to minimize their effects. The calculation of large-scale eddy currents in conductors is an engineering challenge, however. To seek an effective approach for the computation of 3D eddy currents at low frequencies, this study attempts to develop a numerical methodology based on the FDTD technique[1]. In low-frequency and high resolution cases, the former proposed low-frequency schemes [2-4] have difficulty in solving eddy current induction problems in materials with high conductivities due to both the weak coupling between H- and E-fields and the surface wave penetration behavior near conductor-air interfaces. Here we have developed a modified FDTD scheme for eddy current computation in cylindrical conductors.

FDTD Method

By applying the central-difference method with "Leap-frog" fashion introduced by Yee [1] to Maxwell's curl equations in cylindrical coordinates, we derive the update formulations for all the EMF components. The arrangement of the EMF components in the cells is shown in Fig.1. Sources are introduced by the applied current in the coils. The absorbing boundary condition for the termination of the FDTD region is adapted from Berenger's PML which was originally in Cartesian coordinates[1]. To remove the coordinate singularities associated with the axis $r=0$, a series-expansion method [5] was employed to handle the three field components $H_{r0}, E_{f\,0}, E_{z0}$ (see Fig.1). A special polynomial expansion in the radial direction is invoked to ensure a well-behaved solution near the polar axis. The component $E_{z(0,j,k+1/2)}$ can be formulated by considering Amperes law, through a closed circular integral path of radius $\Delta r/2$ around the polar axis. Because of the large conductivity contrast, fields across conductor/air boundaries are rather discontinuous. To incorporate surface wave behavior near the conductor boundary, the surface impedance boundary condition (SIBC)[1] is introduced. Once the fields on the boundary have been obtained, the interior fields can then be updated. Note that the SIBC is formulated in the frequency-domain, which has to be transformed into the time-domain for FDTD implementation using Prony's method [1]. For the harmonic case, the SIBC can be expressed by constant surface resistance and reactance and can be updated in time domain. At low frequencies, Maxwell's equations lose their symmetry, in that the value of $|\sigma/\omega\varepsilon|$ could be of the order of 10^{16}, if the frequency is in the order of hundreds of Hertz or so. This is the case of a good conductor and can be a problem for conventional FDTD methods as displacement currents normally play a role in the iteration procedure. Consider the Helmholtz equation: $\nabla^2\mathbf{H}-\gamma^2\mathbf{H}=0$, where γ is the complex propagation constant. For good conductors, $g = a + jb = \sqrt{wms/2}(1+ j)$. Given a field at the surface of a good conductor, we expect a FDTD solution for the fields that decay away from the surfaces. If we, for example, linearly-scale some of the parameters (ω,μ,σ) , but ensure that the attenuation α and phase constants β are unchanged, the numerical solution should then provide accurate EMFs in the conductors. To do this, we attempt to scale down σ and scale up ω with the same factor. The propagation constant is unchanged and the FDTD formulation can be easily updated. In this way, the wave number $k\approx\sqrt{-j\omega\mu\sigma}$ in the conductor also maintains its correct value. The physical EMFs can be calculated by applying the appropriately inverted scale factors.

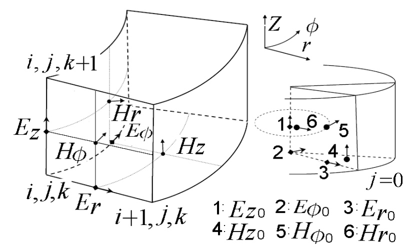

Fig.1. A FDTD lattice and degenerate cells adjacent to the z-axis in cylindrical coordinates. The close loop shows the integral path to solve the singular Ez component in azimuthal direction. E_ϕ, H_r are evaluated by serious expansions.

Simulations

The first simulation considered the eddy currents induced in an infinite cylinder by a circular filamentary current source, driven by a harmonically varying current. The eddy currents have an analytical solution expressed in terms of modified Bessel functions [6]. The numerical results are compared with an analytical solution as shown in Fig.2(a) and indicate the accuracy of the method. The 2nd case studied an MRI magnet model. Here we initially consider only a single cylindrical conductor wall, the magnet bore for eddy current generation (see Fig.2(b)). The dashed cylinder with limited length in the center represents the position of the human body. The EMF source is a Y-gradient coil designed using a simulated annealing method [7]. The calculated H- and E-field patterns are shown in Fig.2(c) and (d), which depicted the field decay behavior inside the metals. Also, the numerical result for gradient fields Bz profile (see Fig.2(c)) indicates a very linear gradient over the region of interest.

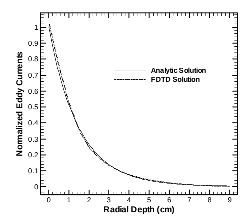

Fig.2(a)

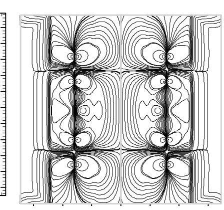

Fig.2(c)

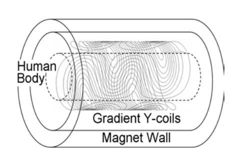

Fig.2(b)

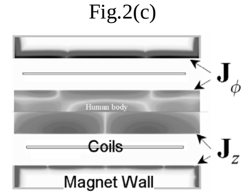

Fig.2(d)

Fig.2. The implementation of the FDTD algorithm. (a): comparison with the analytic solutions. The eddy current along the radial direction inside a cylinder tube excited by concentric current loop; (b) Cylindrical metal wall excited by a Y-gradient coil in an MRI environment; (c) The contour of the Bz in the z-r middle plane; (d) The contour of the eddy currents in the z-r middle plane.

Conclusion

Preliminary application of an FDTD method to 3-D eddy current problems in MRI applications has been presented. Several unique features of the FDTD variant have allowed its application to this difficult problem. By enabling an FDTD method for this work we are in a position to approach a complete model of an MRI system, including tissue-field interactions and the effect of pulsed field gradients on surrounding conductors.

References

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