# Optimized diffusion tensor encoding schemes with anisotropic diffusion weighting

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**Background/Introduction**

Diffusion tensor MRI and the associated measures, such as FA, trace {D}, and eigenvector direction, are highly sensitive to image measurement noise. The main strategy to decrease noise sensitivity is to employ uniformly distributed diffusion encoding directions with a diffusion-weighting value that is nearly optimum for the average diffusivity (ADC) [1-4]. These approaches make sense for the case where the diffusion tensor distributions and directions are arbitrary or unknown. However, in the case where the diffusion tensor shape and orientation in a specific region may be estimated a priori (such as in either the corpus callosum, the corticospinal tract or the spinal cord), it may be possible to make more precise measurements in that region by using an anisotropic diffusion-weighting scheme. In this study, the diffusion-weighting was optimized for each encoding direction to minimize the error in FA measurements of corpus callosum.

**Theory**

The fractional anisotropy (FA) is a commonly anisotropy measurement, and it is defined as $FA = \sqrt{\frac{3}{2}}\sqrt{\frac{(\lambda_1 - <\lambda_i>)^2 + (\lambda_2 - <\lambda_i>)^2 + (\lambda_3 - <\lambda_i>)^2}{(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)}}$ , where $\lambda_i$ are the eigenvalues of a diffusion tensor. Since the tensor elements are constructed from directional diffusitivities $D_i (= \frac{\ln So - \ln S_i}{b_i})$ , it is possible to derive the expression of FA in terms of $D_i$ , which leads to an expression of FA variance. $\sigma_{FA}^2 = \sum_i^n (\frac{\partial FA}{\partial D_i})^2 \sigma_{D_i}^2 + 2\sum_{i \neq j} \frac{\partial^2 FA}{\partial D_j \partial D_i} \sigma^2 D_i D_j$ [eqn 1]. Powell's method [5] was used to optimize the directional diffusion weighting. The diffusivity variances and covariance terms are as follows: $\sigma_{D_i}^2 = \frac{\sigma_{S_0}^2}{S_0^2} \frac{(1 + \exp(2 b_i D_i))}{b_i^2}$ [4] $\sigma^2 D_i D_j = \frac{\sigma_{S_0}^2}{S_0^2} \frac{1}{b_i b_j}$

**Methods**

A single-shot spin echo EPI sequence with diffusion-tensor encoding (12 directions (optimized using minimum energy criterion [3]), & b = 1000s/mm$^2$) was used to estimate the diffusion tensor of the corpus callosum. The corpus callosum was selected by use of a manual ROI and selecting voxels FA > 0.6 and the x component of the major eigenvector was > 0.9. The measured diffusitivities of these voxels were averaged for each encoding direction to estimate a representative set of directional diffusitivities in the corpus callosum . Powell's optimization method was used to estimate the 12 directionally optimum b factors by minimizing the variance of FA. The optimum diffusion-weighting for each direction is listed in Table 1. Note that the optimum directional diffusion weightings ranged between 595 and 2014 s/mm$^2$. In order to achieve the necessary diffusion-weighting for the anisotropically optimized encoding set, $\Delta$ (interval between the two diffusion gradient pulses), $\delta$ (the diffusion gradient pulse duration) were increased from 21 ms to 26.2 ms and from 27.4 ms to 32.2 ms, respectively. The error propagation predicted that the variance in FA should decrease by 56% by using the anisotropic diffusion-weighting. A subsequent diffusion tensor scan was performed on the same subject using both with the isotropic diffusion-weighting (b = 1000 s/mm$^2$) and the optimized anisotropic diffusion-weighting scheme listed in Table 1. The scan was repeated nine times for each set of encoding weightings to estimate the variance in FA for the corpus callosum.

**Results**

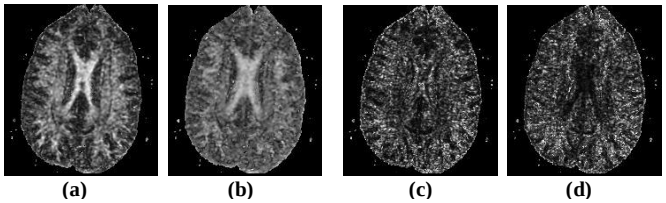
Axial FA maps through the body of the corpus callosum for isotropic diffusion weighting and the optimized anisotropic diffusion-weighting are shown in Figure 1 a and b, respectively. In general, the corpus callosum appears fuller and less noisy in the FA image obtained with anisotropic diffusion-weighting (Fig 1b). The other white matter regions, however, appear blurrier and noisier). The variances of the FA maps across the 9 runs are shown in Figure 1 c and d. The variance in the corpus callosum is lower for anisotropic diffusion-weighting (Fig. 1d) but higher in most other brain regions. Plots of the measured FA values in one voxel for the 9 runs are shown in Fig 1 e and f. The variance in the FA for the anisotropic diffusion weighting is significantly lower by a factor of 12.8 in the voxel. The overall reduction in the corpus callosum region using optimum b factors was threefold. The optimum tensor encoding was applied to four other subjects and similar results were obtained. In addition to the reduction of FA variance, new optimum b factors decreased the residual error $\sum_i \left| D_i (= \frac{\ln So - \ln S_i}{b_i}) - g_i^T \overline{D} g_i \right|$ in the corpus callosum region from ~2.5 to ~2.1. This fact ensures a better fitting to the single tensor model with the optimum b factors.

**Discussion**

Optimizing the diffusion-weighting for individual encoding directions was found to reduce the variances of FA measurements in regions where there was an a priori estimate of the apparent water diffusion tensor. However, in regions where the diffusion tensor was not similar to the optimization case, the accuracy tended to be similar or worse. The sensitivity of the anisotropic encoding to slight variations in tensor shape and orientation is still unknown and will be the focus of future studies. However, the optimization from one subject in this study appeared to work well in scans on four other subjects. Although not shown, the method also reduced the variance in the tensor trace and individual diffusivities. In addition to the corpus callosum, this method may be useful for studies of other white matter regions that are relatively homogeneous and the approximate direction is known before the experiment, such as the spinal cord.

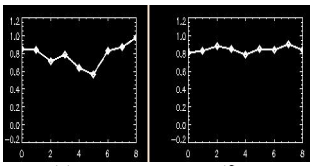
**TABLE 1.**

| Encoding direction | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Optimum b factors | 1759 | 1033 | 1968 | 825 | 2014 | 1954 | 510 | 872 | 595 | 1104 | 1971 | 623 |



(a) (b) (c) (d)

Figure 1(a): the conventional FA map using b= 1000 s/mm$^2$
Figure (b): FA map using optimum b factors listed in the Table 1
Figure (c): the variance map of FA over 9 times repeated measurements using conventional b factors
Figure (d): the variance map of FA using optimum b factors.



(e) (f)

Plot 1 (e): a temporal variation from a pixel in the CC of figure (a)
Plot (f): a temporal variation from the same pixel in the CC of figure (b)

**References**

[1] Papadakis N.G et al JMR 1999; 137:67-82. [2] Jones, D.K. et al MRM 1999; 42:515-525. [3] Hasan K.M. et al JMRI 2001; 13:769-780. [4] Xing et al MRI 1997; 15:771-784. [5] William H. Press Numerical recipes in C: The art of scientific computing