MRI Image Reconstruction by Polar Fourier Transform

H. Guo[1], A. W. Song[1]

[1]Brain Imaging and Analysis Center at Duke University, Durham, NC, United States

Abstract:

Polar Fourier Transform (PFT) has been used for applications in which data is sampled on polar grids, such as in radar imaging[1]. However it has not seen wide use in MRI because most of the images are acquired in Cartesian coordinates. For k-space data sampled on non-Cartesian grids, such as those using radial and spiral sampling schemes, PFT may provide a simple expression for image reconstruction. We demonstrate, in this report, using k-space data sampled concentrically over polar grids, the advantage of using PFT for image reconstruction to achieve low estimation error. In addition, a lookup table can be incorporated to drastically reduce the computation time.

Introduction:

For k-space data sampled over polar coordinate grids, conventional FFT cannot be applied directly without resampling the data points back to the Cartesian grid. This re-gridding procedure often introduces estimation errors to the final result. A method which avoids this intermediate interpolation step was proposed by Higgins et al.[2] for Computed Tomography (CT) image reconstruction. In their implementation, a Hankel Transformation was used to achieve the Polar Fourier Transform (PFT) for the projection data.

We developed and applied PFT to MRI image reconstruction using non-Cartesian sampling. The k-space data was acquired along circular symmetric trajectories. Traditionally, such data are reconstructed using back-projection or re-gridding (interpolation) methods. In this report, the feasibility and performance of the PFT method was tested by computer simulation, as well as real data.

Method:

The continuous Cartesian Fourier transform is given by $F(K_x,K_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\exp(-j(k_x x + k_y y))dxdy,$ *where* k_x and k_y represent frequency domain, x y spatial domain, and f(x, y) is an arbitrary continuous function. To convert it to polar system, let $x = r\cos\theta$, $y = r\sin\theta$, $k_x = \rho\sin\phi$, $k_y = \rho\cos\phi$, where r and θ are in spatial domain, ρ and ϕ in frequency domain. Then the PFT can be expressed as: $F(\rho,\phi) = \int_0^{\infty}\int_0^{2\pi} rf(r,\theta)\exp(-j\rho r\sin(\theta+\phi))d\theta dr$, (1).Since $f(r,\theta)$ is periodic in θ with 2π, it can be written into Fourier series, $f(r,\theta) = \sum_{-\infty}^{\infty} C_n(r)\exp(jn\theta)$, where $C_n(r) = 1/2\pi\int_{-\pi}^{\pi} f(r,\theta)\exp(-jn\theta)d\theta$, the Fourier coefficients of $f(r,\theta)$ at radius r. So (1) can be written as: $F(\rho,\phi) = 2\pi\sum_{-\infty}^{\infty}\exp(-jn\phi)\int_0^{\infty} rC_n(r)J_n(\rho r)dr$, (2), where $J_n(x) = 1/2\pi\int_0^{2\pi}\exp(j(n\theta - x\sin\theta))d\theta$. From the n-th order Hankel Transform (HT) of a function f(r) $F_n(\rho) = H_n\{f(r)\} = \int_0^{\infty} rf(r)J_n(\rho r)dr$, and its inverse transform $f(r) = \int_0^{\infty}\rho\overline{F_n(\rho)}J_n(\rho r)d\rho$. (2) can be expressed as $F(\rho,\phi) = 2\pi\sum_{-\infty}^{\infty}\exp(-jn\phi)\overline{C_{nn}(\rho)}$ where $\overline{C_{nn}(\rho)} = \int_0^{\infty} rC_n(r)J_n(\rho r)dr$ is HT of $C_n(r)$.

The polar grid data $F(r,\theta)$ is first reshaped as a N_α x N_r matrix with N_α and N_r representing azimuth and range direction sampling number respectively. Then FFT is applied in azimuth direction to get $C_n(r)$. Next discrete Hankel Transform is used to calculate $C_{nn}(\rho)$. The final step is to do FFT of $C_{nn}(\rho)$ to $f(\rho,\phi)$. This is the final image in polar coordinate. The image can also be gotten from $F(r,\theta)$ through direct summation, but it is not computation efficient. The total computation complexity for PFT is $O\{N_r N_\alpha \log_2 N_\alpha\} + 0.5O\{N_r N_\alpha \log_2 N_r\} + O\{N_r N_\alpha \log_2 N_\alpha\}$. For the conventional regridding method it is $O\{N_x N_y\} + O\{N_x N_y \log_2 N_x N_y\}$.But $J_n(\rho r)$ can be pre-calculated to improve the computation speed. Computer simulation data and real radial sampled data was reconstructed using PFT and the conventional regridding method to evaluate the reconstructon quality of PFT.

Result:

The resulting images from PFT were interpolated onto Cartesian coordinates from Polar system for display. Figure 1 shows the computer simulation results, demonstrating that PFT has less estimation errors than the re-gridding method, which can be seen from the central line profile plot in figure c. On a PC with a 2.4 GHz CPU, the computation took 54.52 seconds for PFT when the n-th order Bessel function of the first kind was calculated directly, and the re-gridding method took 4.51 seconds. If the Bessel function was saved as a lookup table, the entire computation for PFT was shortened to 0.52 seconds. For the real data from the radial sampling, results are shown in figure 2. Lower estimation error is again achieved with PFT.

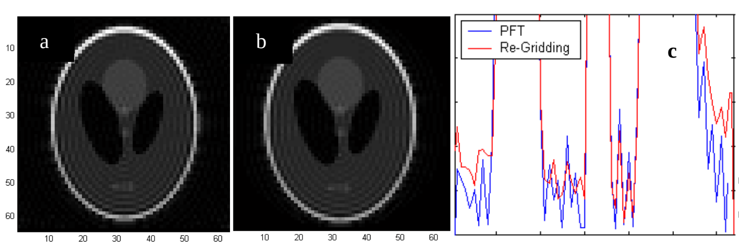

Figure 1. Computer simulation. a), gridding; b), PFT, c), central line profile plot.

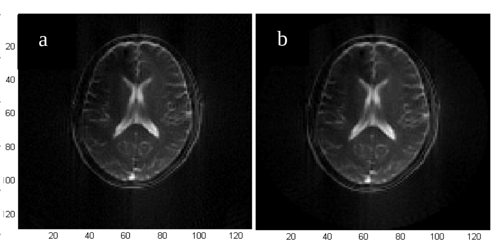

Figure 2. *In vivo* data reconstruction. a), gridding; b), PFT.

Discussion:

Both computer simulation and real data showed that PFT improved spatial accuracy in the reconstructed image. In addition, this method does not require a density compensation function to be applied in k-space, which is routinely used in the conventional re-gridding algorithm. The Bessel function J_n can be prepared as a lookup table for the same k-space trajectory data to drastically improve the computation speed. Further improvements can be achieved still by adopting fast HT[3,4]. Given the increased use of spiral imaging, this algorithm can reconstruct spiral images by splitting the spiral k-space data into segments that approximate the circular symmetry. Such effort is currently underway in our lab.

Reference:

1. J. Laar et al., *Proc. ProRISC*, ISBN: 9073461243 (Netherlands), 2000. **2**. W. E. Higgins et al., *IEEE Trans. Med. Imag.*, 7:59, 1988. **3**. M.J. Cree et al., *Comp. Math. Applic*. 26: 1-12, 1993. **4**. L. Knockaert, *IEEE Trans. Sig Proc.* 48: 1695 –1701, 2000.