

Efficient iterative reconstruction for MRI in strongly inhomogeneous B₀

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INTRODUCTION:

In MRI, inhomogeneity of the main magnetic field B₀ causes significant shifts and distortions when the field variations are large compared to the pixel bandwidth. This is frequently the case at high field strength and with long read-out trains such as in echo-planar and spiral or high resolution imaging. At the reconstruction level, the problem of B₀ inhomogeneity is typically addressed by unwarping (1) or conjugate phase reconstruction (2). The latter can be performed efficiently with time segmentation (3) or multifrequency interpolation (MFI) (4). However, the conjugate phase approach relies on the assumption that B₀ varies smoothly in space. Recently a more general, iterative solution was suggested based on the conjugate gradient (CG) algorithm (5). This method does no longer rely on smooth B₀, yet it is limited by its substantial numerical complexity. In the present work we propose several modifications of the CG approach, addressing the efficiency and robustness of the procedure, as well as the optimality of the spatial response function (SRF).

THEORY AND METHODS:

The problem of compensating for in-plane B₀ inhomogeneity in the reconstruction is frequently badly conditioned, leading to numerical instability and local noise enhancement. This issue can be addressed by regularization, i.e. by judiciously accepting a certain level of error in signal reconstruction in turn for stabilization. For balancing the two competing goals, we propose to minimize the weighted sum of the noise variance and the expected squared signal error on a pixel-by-pixel basis. This is achieved with the following reconstruction formula:

$$\mathbf{I} = \beta \theta \mathbf{E}^H (\beta \mathbf{E} \theta \mathbf{E}^H + \alpha \Psi)^+ \mathbf{m} \quad (1)$$

where \mathbf{m} is a vector of the measured data in k-space, \mathbf{I} the vector of the reconstructed pixel values, \mathbf{E} the encoding matrix and \mathbf{E}^H its Hermitian adjoint. The superscript + denotes the Moore-Penrose pseudoinverse. α and β are coefficients representing the weight of noise and signal variance, respectively, in the joint minimization. Ψ denotes the noise covariance of the sample values and θ denotes the signal covariance, which represents potential prior knowledge about the signal distribution in the object. No prior knowledge corresponds to θ equal to identity. Note that the set of target pixel positions can be arbitrarily large, corresponding to an arbitrary target resolution. In the limit of high target resolution, $\alpha = 0$ and $\theta = \text{identity}$, Eq. (1) yields for each pixel that SRF which best approximates a Dirac function in the least-squares sense. For the particular case of Fourier encoding under the influence of B₀ inhomogeneity, the encoding matrix reads

$$\mathbf{E}_{\kappa,p} = \exp(-i\mathbf{k}_\kappa \mathbf{r}_p - i\omega_p t_\kappa), \quad (2)$$

where \mathbf{k}_κ and \mathbf{r}_p denote the κ -th sampling position in k-space and the position of the p -th pixel in the image domain, respectively. ω_p denotes the B₀-induced frequency offset at the position \mathbf{r}_p , and t_κ denotes the time at which the sample κ is taken. For image reconstruction with Eq. (1), the matrix inversion part is solved by the CG method. Its solution \mathbf{x} can then be pre-multiplied by the left-most part of (1) to get the reconstructed image: $\mathbf{I} = \beta \theta \mathbf{E}^H \mathbf{x}$. The efficiency of each loop in the CG scheme can be enhanced considerably by performing the matrix-vector multiplications with \mathbf{E} and \mathbf{E}^H by combining MFI (4) with fast Fourier transform, in a scheme related to an approach proposed in Ref. (6) (see Fig.1).

Experiments were performed on a 3T Gyroscan Intera system (Philips Medical Systems, Best, The Netherlands), imaging a water bottle with a head coil. To induce substantial B₀ inhomogeneity, two small permanent magnets were placed at the phantom's surface, creating B₀ offsets ranging from -700Hz to +2860Hz. For creating strongly B₀-affected data, a spin echo technique with very small pixel bandwidth of 43 Hz was used with a matrix of 256x256. B₀ maps were computed from two GRE images obtained with different echo times. The resulting map was unwarped (1), smoothed by polynomial fitting and linearly extrapolated (Fig.2). Then the fringes of the expanded B₀-map were pushed to zero. α and β were chosen such as to yield approximately identical noise and artifact levels. θ was set to a diagonal matrix, with ones for pixels inside the object and entries tailing away to zero outside the object. Since the samples were taken at different times, noise was uncorrelated and Ψ was a multiple of identity, whose scaling could be assessed from the image background.

RESULTS:

The measured data was reconstructed using both a conventional MFI and the proposed reconstruction (6 iterations, reconstructed on a 512x512 grid, 5 min computation a 2.8 GHz CPU) (see Fig. 3). Both methods nearly recover the original shape of the phantom. The main advantage of the proposed reconstruction is that it correctly distributes the signal within the object and does not assign excessive signal close to the region of strongest B₀-variation. The remaining slight distortion is due to small error in B₀ assessment, which is most likely related to different eddy currents between B₀ mapping and the spin echo sequence. This was verified by simulated data acquisition from a disk object based on the measured B₀. The subsequent reconstruction was indeed free of residual distortion (see Fig.4).

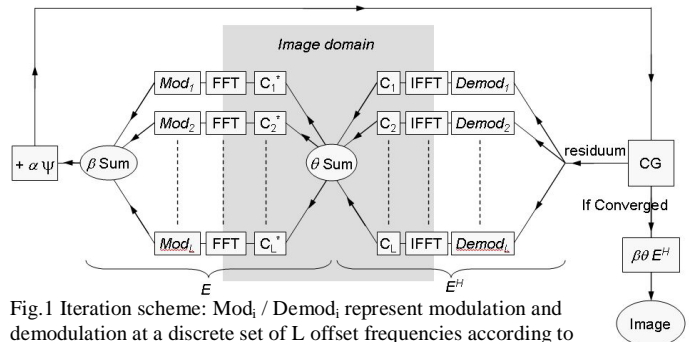
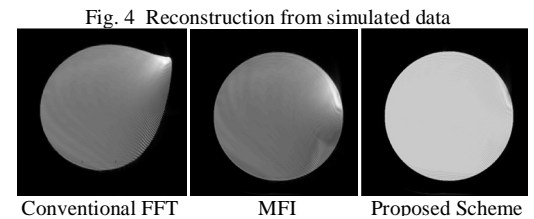
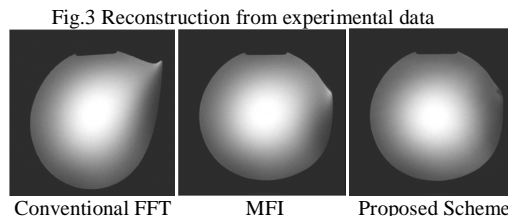
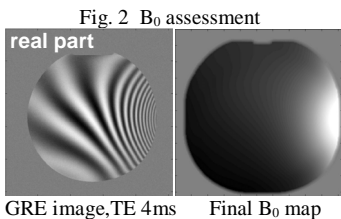


Fig.1 Iteration scheme: Mod_i / Demod_i represent modulation and demodulation at a discrete set of L offset frequencies according to MFI. C_i denote multiplication by the corresponding coefficient maps.

DISCUSSION AND CONCLUSION:

The proposed method permits efficient B₀-corrected reconstruction without restrictions in terms of local B₀ variation. The reconstruction scheme is readily applicable to EPI data if the k-space trajectory is accurately known, suggesting promising applications in fMRI and DTI. Ultimately, the reconstruction is limited by the inherent information content of the acquired data. Excessive inhomogeneity effects will cause deteriorating conditioning, shifting the balance in the weighted minimization towards unfavorable compromises of the spatial response. Potential means of working against this problem are parallel acquisition with coil arrays and fast, non-Cartesian sampling patterns. Both options can be readily accommodated in the proposed framework by modifying the encoding matrix and the iteration scheme (6).

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