Beyond the g-factor limit using joint entropy

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Introduction: Partially Parallel Imaging (PPI) can reduce the number of phase encoding steps required when using array coils, resulting in speedup, but it may also lead to noise amplification in the final image if the coils do not provide sufficiently spatially independent data. Such noise amplification (g-factor effect[1]) is recognised as a limiting factor for the maximum speed-up that can be achieved in practice[2]. Various methods for reducing g-factor enhanced noise have been proposed, including adaptive averaging[3] and regularised reconstruction methods. These methods decrease noise fluctuations at the expense of spatial smoothing or potentially introduce bias from the prior information applied e.g. resolution reduction and/or change in contrast. In this study we have investigated the use of joint Entropy to reduce g-factor effects in PPI. Single image entropy has previously been used to explore this problem with limited success due to the limited coherent structure remaining in badly artefacted images[4].

Theory: For two images A and B, the joint probability p(a,b) of intensities a in image A and b in image B occurring together can be used to define a joint entropy (E_j), defined as the sum over all p(a,b) of p(a,b)log p(a,b). E_j is a measure of the degree to which image B predicts the intensity in image A and vice versa[5]. In this application we take image A to be a PPI reconstructed image and image B to be a reference image, the properties of which we will discuss later. The amplified noise in A reduces the shared information content and increases E_j. For regularly undersampled data, the SENSE method can be used for PPI reconstruction. In the image domain aliased pixels are separated using linear algebra into regularly spaced families of final image pixels. If the corresponding aliased pixels for each coil are assembled in to a complex vector S and the unfolded pixels are written as a complex vector X, then X = CS, where C is a (complex) reconstruction matrix that is derived from the coil sensitivities and may take account of any noise correlations. In practice noise is present in the images, here we consider only noise in the target images S, then we have S = S' + dS, where dS is a noise vector and S' is an ideal noise free signal. The amplified noise in the final image is given by dX = CdS. When the g-factor is high, C is ill conditioned so there tends to be a single dominant eigenvector. We can now approximate the elements dX by $ds.\alpha\lambda$., where α is the principal eigenvector and λ its eigenvalue and ds is a single unknown complex scalar that is to be determined by minimising E_j.

Method: The method was tested using synthetic images from the MNI brain phantom[6]. The images were multiplied by candidate coil profiles before being transformed to k-space, sub-sampled and then reconstructed again as aliased images. Independent complex Gaussian noise was added to each aliased coil image and a SENSE reconstruction performed. Tests were performed with a linear speed-up factor of four using a model linear array of four coils with real and Gaussian sensitivity profiles chosen to have width 12.5cm and separation 4cm to exacerbate g-factor noise enhancement. The worst-case signal to noise ratio of approximately 1.5 in the unfolded image compared to 100 in the starting single coil images. The reference images were also derived from the MNI brain phantom. The pixels for correction were selected using a threshold on the determinant of the matrix C. This process selected only those pixels with a true degeneracy of 4 in this case. The coil sensitivity data was acquired using a combination of surface coil images and a body coil image. The joint Entropy was calculated between the body coil reference image and the SENSE image modulated by the body coil profile.

In general E_j is constant for many values of ds except those very close to the true value, this reflects the sparseness of the joint histogram and prevents the uses of gradient driven optimisation routines. Minimisation of E_j was achieved using a simple search strategy for ds for each group of aliased pixels where the bounds of the search space were set based on knowledge of the noise in the aliased images. All code was written in IDL and computations were performed on a Compaq Alpha station. The following trials were performed, in each case the reference image was used for both the SENSE processing and the subsequent correction:

1) A = T2w image, B = identical T2w image. 2) A = T2w image with simulated rectangular lesion, B = T1w image with no lesion. 3) A = T2w image with MS lesions, B = low resolution T1w image with lesions. Results were visually assessed and assessed by subtraction of the known gold-standard image to detect residual structure that might indicate image modification.

Results: In all cases the noise in the PPI image was substantially reduced often to less than that in the neighbouring regions. Lesions were preserved even when not present in the reference data and there was no evidence of a change in image contrast or resolution. The residual signals remaining after subtraction of the gold-standard image was consistent with Gaussian noise (albeit quantised by the step size set in the search strategy) and did not display signals related to the underlying image structure.

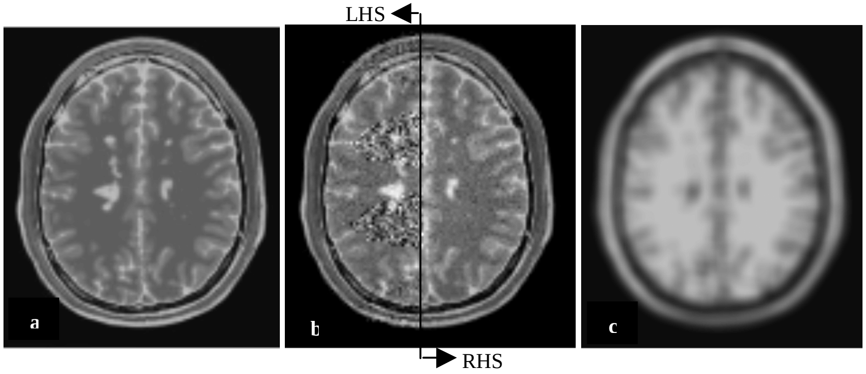

Figure 1: a) original gold-standard noise free image from MNI, b) LHS, image after PPI speed-up factor 4 with simulated 4 coil array, RHS PPI image after minimisation of E_j and c) low resolution T1w image used as reference image for both PPI and Ej minimisation. This example best reflects actual practice where the reference data is likely to be of different resolution and contrast to the target data. Note that the lesions present in the gold image (which were obscured by noise in the original PPI reconstruction) are now visible and that they are not visible in the reference data due to contrast and resolution differences. The correction has been applied only to the truly 4 fold degenerate "shark tooth" shaped regions.

Discussion and conclusion: Joint entropy provides a robust means of removing g-factor amplified noise without modifying intrinsic image properties. The reference image provides information only about the likely distribution of tissue classes in the damaged image and so does not need to be the same contrast or resolution. Unlike conventional regularisation schemes, the use of joint entropy does not lead to bias in the final result. A key element of this method is the use of the principle eigenvector of C, which constrains the allowed distributions of estimated noise across the degenerate pixels. This constraint ensures that minimisation of E_j does not change tissue classes of pixels in the PPI image, even in the presence of tissue unclassified in the reference image. The ability to apply this method using low resolution reference images indicate that the reference data usually collected to determine the coil sensitivities could also serve as a reference for the anatomy.

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References:

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