The influence of membrane permeability on bi-exponential behavior of diffusion-attenuated MR signal from a single cell

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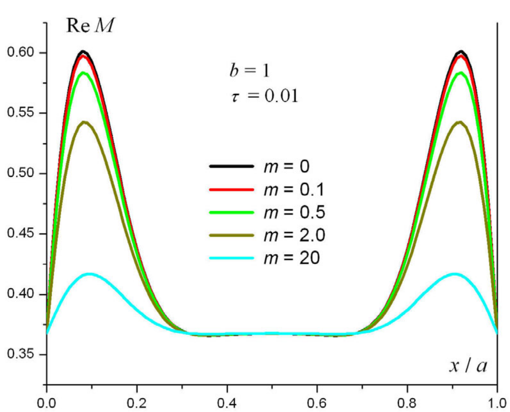
We recently analyzed in detail the behavior of the diffusion-attenuated MR signal in a single compartment with impermeable boundary (1). It was shown that for short diffusion time $\Delta = t_D$, $t_D = a^2/D$ is the characteristic diffusion time (*a* is a compartment size, *D* is the free-diffusion coefficient), the signal dependence on the *b*-value can be approximated to a remarkable degree by the bi-exponential function $S = f_1 \exp(-bD_1) + (1-f_1)\exp(-bD_2)$. The physical underpinning of this approximation is the presence of a strongly inhomogeneous distribution of magnetization at $\Delta = t_D$. Here all spins can be conditionally divided into two populations: one population, comprising spins located far from the boundary, can be considered as unrestricted (fast-diffusing pool); the other population, comprising spins located near the boundary, is restricted due to encounters with the boundary (slow-diffusion pool). The subject of this communication concerns the manner in which a boundary with finite permeability μ "nibbles" at the slow-diffusing pool and affects the bi-exponential behavior of the signal.

For short diffusion times, the MR signal from a single compartment of a size *a* is similar to the signal from a one-dimensional periodic structure of period *a*, which allows an exact analytical solution. In the narrow pulse approximation, we find rather simple expressions for the local magnetization distribution inside the compartment $M(x,\Delta)$ and the net spin echo signal $S(\Delta)$:

$$M(x,\Delta) = \sum_n \frac{2iq\exp(-iqx)\cdot\exp\left(-\kappa_n^2\tau\right)\kappa_n\left[\cos\left(\kappa_n(1-x/a)\right)-\exp(iq)\cos\left(\kappa_n x/a\right)\right]}{\left((qa)^2-\kappa_n^2\right)\left((2m+1)\sin\kappa_n+\kappa_n\cos\kappa_n\right)}, \qquad [1]$$

$$S(\Delta) = \frac{2q^2}{m}\sum_n \frac{\exp\left(-\kappa_n^2\tau\right)\kappa_n^2}{\left((qa)^2-\kappa_n^2\right)^2\left(2m+1+\kappa_n\cot\kappa_n\right)} \qquad [2]$$

where $q=\gamma G\delta$, γ is the gyromagnetic ratio, *G* is a field gradient, δ - is a pulse duration, $\tau = \Delta/t_D$, $m = \mu a/D$, κ_n are non-negative roots of the equation $2m\left(\cos\kappa - \cos(qa)\right) = \kappa\cdot\sin\kappa$. In the limit of completely permeable membranes (free diffusion, $m=\infty$), Eq. [1] reduces to the standard homogeneous magnetization $M(x,\Delta) = \exp(-Dq^2\Delta)$. In the opposite limit of completely impermeable membranes, Eq. [2] reduces to the well-known result (2).



As an example, the real part of the function $M(x,\Delta)$ is shown in Fig. 1 for several values of the dimensionless permeability m (time Δ and the *b*-value $b=(qa)^2\Delta$ are fixed as indicated). Note that for $m \le 0.1$ the magnetization distribution practically coincides with the case of a completely impermeable boundary. The two clearly distinguished maxima in *M*, making up the slow-diffusing pool, decrease as m increases and it should be expected that the apparent volume fraction of this pool f_1 also must decrease. Modeling the net signal $S(\Delta)$ as a bi-exponential function shows that as m increases f_1 decreases from ~10% at m=0, to 8% at m=1 (typical of mammalian cells), to 1% at m=20, the apparent diffusion coefficient D_2 of the fast-diffusing pool approaching *D*, and D_1 tending to 0.44 *D*.

We conclude that bi-exponential signal behavior predicted at short diffusion time $\Delta = t_D$ for a single compartment with impermeable boundary [1] is still found in the presence of a boundary with finite permeability in the range expected of biological membranes.

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1. Sukstanskii, A., Ackerman, J.J.H., and Yablonskiy, D.A. Magn. Reson. Med. 2003;55:735-742.
2. Tanner, J.E., and Stejskal, E.O. J.Chem.Phys. 1968;49:1768-1777.