A Constrained Minimization Approach to Designing Multi-dimensional, Spatially Selective RF Pulses

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Introduction

The design of multi-dimensional, spatially selective RF pulses can be viewed as analogous to image reconstruction in many ways [1]. It shares many of the technical challenges in reconstruction such as the problems of main field inhomogeneity and density compensation function definition, but it also differs in important ways. For example, there are hardware and SAR constraints on RF pulses, and excitation patterns outside object (or region of interest) can be undefined. This article presents a new approach to RF pulse design using constrained minimization. This new approach can be used to address many RF pulse design issues.

Theory

Our design approach is based on the concept of excitation k-space [2]. For small tip angles, the transverse magnetization $m_{xy}(\mathbf{x})$ produced by an RF pulse can be approximated by

$$m_{xy}(\mathbf{x}) \approx i\gamma m_0 \int_0^T b_1(t) e^{i\omega(\mathbf{x})(t-T)} e^{i\mathbf{x}\cdot\mathbf{k}(t)} dt, \qquad \mathbf{k}(t) = -\gamma \int_t^T \mathbf{G}(\tau) d\tau \quad (1)$$

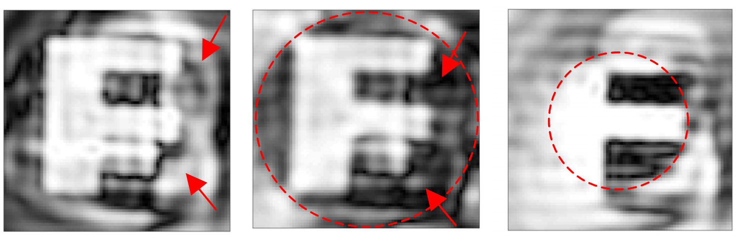
where $\mathbf{k}(t)$ is any legitimate excitation k-space trajectory defined by gradients $\mathbf{G}(t)$, $b_1(t)$ is the complex RF envelope, γ denotes the gyromagnetic ratio, m_0 the equilibrium magnetization, and $\omega(\mathbf{x})$ the resonance frequency offset due to main field inhomogeneity. We discretize (1) by uniformly sampling M points in space and N points in time, leading to $\mathbf{m} \approx \mathbf{A}\mathbf{b}_1$, where $\mathbf{m}$ and $\mathbf{b}_1$ are the sample vectors, and $\mathbf{A}$ is an $M \times N$ matrix with $a_{m,n} = i\gamma m_0 \exp[i\omega(\mathbf{x}_m)(t_n - T) + i\mathbf{x}_m \cdot \mathbf{k}(t_n)]\Delta t$, in which Δt is the RF temporal sampling period. Thus, given a predetermined trajectory, measured field map, and a M-sample desired magnetization pattern $\mathbf{m}_{des}$, one can view the design of a multidimensional spatially selective RF pulse as a constrained minimization problem:

$$\tilde{\mathbf{b}}_1 = \arg\min_{\mathbf{b}_1} \|\mathbf{A}\mathbf{b}_1 - \mathbf{m}_{des}\|_{p,W} \quad \text{subject to} \quad \|\mathbf{b}_1\|_2^2 \le C_1 \quad \text{and} \quad |b_{1,i}|^2 \le C_2, \quad i = 1,\ldots,N, \quad (2)$$

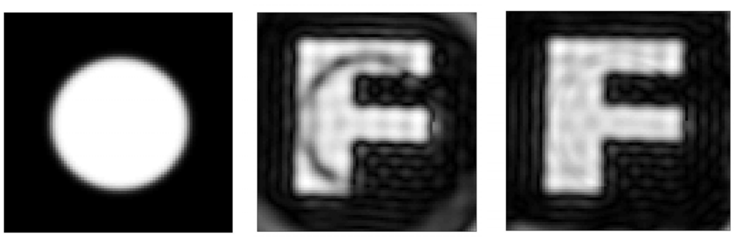
where $\|\cdot\|_{p,W}$ denotes the p-norm weighted by W, which is a $M \times M$ matrix specifying spatial excitation error weighting, C_1 and C_2 are constants, and the two constraints are the specific absorption rate (SAR) and peak RF magnitude constraints respectively. For $p = 2$, we can obtain an approximate solution to (2) using iterative methods (such as preconditioned conjugate gradient) on the following penalized least-square problem [3,4]:

$$\tilde{\mathbf{b}}_1 = \arg\min_{\mathbf{b}_1} \left[\|\mathbf{A}\mathbf{b}_1 - \mathbf{m}_{des}\|_{2,W}^2 + R(\mathbf{b}_1) \right], \quad (3)$$

where $R(\mathbf{b}_1)$ is a regularization term for satisfying the two constraints. Details of this regularization term will be presented in other works by the same authors.



(a) With conjugate-phase method using under-sampling trajectory.
(b) With minimization method using under-sampling trajectory. W = entire circular FOV (dashed line).
(c) With minimization method using under-sampling trajectory, W= shrank unit disc (dashed line).



(d) Field map. (in disc: 100 Hz, elsewhere: 0 Hz)
(e) With conjugate-phase method, incorporated with field map in (d).
(f) With minimization method, incorporated with field map in (d).

Method and Results

2D selective excitation simulations were performed with a Bloch-equation simulator, which simulates magnetization in a volume excited by input RF and gradient waveforms. FOV of simulation was 20 cm $\times$ 20 cm, with resolution of 0.25 cm. The excitation k-space trajectory was a uniform-density spiral designed for maximum gradient slew rate 100 T/m/s and peak gradient strength 0.04 T/m. In all simulations, the desired excitation pattern is a character "F" in a FOV of 20 cm $\times$ 20 cm, with 1 cm resolution. We compare our new RF pulse design method to the conjugate-phase (CP) method analogous to the CP reconstruction algorithm, used in [1,5].

The first set of simulations (Fig. (a)-(c)) make comparisons in case of an under-sampling k-space trajectory. The under-sampling spiral was derived for an excitation FOV of diameter 10 cm, with 1 cm resolution. Fig. (a) shows the magnitude of the excitation pattern produced by the CP method. Fig. (b) shows the pattern produced by RF waveform calculated according to (3), with W equals 1 inside a disc of diameter 20 cm, and 0 elsewhere. A preconditioned conjugate gradient algorithm was used for the minimization, and SAR and peak RF magnitude limits were not considered (i.e., R($\mathbf{b}_1$)=0). Our method outperformed the CP method within the circular FOV. Fig. (c) shows the resulting pattern with the same parameters as in Fig. (b), except that the diameter of the unit disc is shrank by 40%. It illustrates that excitation error can be reduced in regions of interest if we sacrifice that of uninterested regions.

The second set of simulations (Fig. (e), (f)) compare performances when field inhomogeneity is present (in Bloch-equation simulator). The trajectory used here is a fully-sampling spiral for an excitation FOV of diameter 20 cm, with 1 cm resolution. Fig. (d) shows the 20 cm$\times$20 cm field map (100 Hz in disc, 0 elsewhere). Fig. (e) shows magnitude of the resulting pattern obtained with the CP method with the field map taken in account. There are "artifacts" in regions where the field map has high gradient (edge of the disc). Fig. (f) shows resulting pattern obtained with our method, incorporated with the same field map .

Conclusion

This constrained minimization approach of RF pulse design has several advantages over conventional methods such as CP: 1. selective excitation performance with under-sampling k-space trajectories can be optimized; 2. weighting pattern W can be used to specify relative excitation precision in different spatial regions; 3. density compensation function is not needed; 4. field inhomogeneity is accounted for without field map smoothness assumption; 5. unrealizable RF waveforms can be avoided via constraints. With this improved design approach, multi-dimensional spatially selective excitation can potentially find an even wider scope of MR applications.

References

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