Parallel Imaging as a Non-linear Inversion Problem - Improved Reconstructions

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Introduction

Parallel imaging promises large scan time reductions. Basically, current reconstruction methods invert a linear system which models the physical generation of the measured electromagnetic signal. This linear system is in part determined by the known k-space trajectory (e.g. the sampling pattern for Cartesian trajectories) and in part by the *a priori* unknown or only approximately known coil sensitivities. Because the latter are influenced by the electromagnetic properties of the object, it is often necessary to use autocalibrating methods which deduce the coil sensitivity information from a few central autocalibration lines acquired during the same measurement as the rest of the data. Using only very few reference lines introduces errors into the coil sensitivity maps which might lead to residual ghosting artifacts in the reconstructed image. On the other hand, using a large number of reference lines is contrary to the goal of reducing scan time. For this reason we identified methods which attempt to better use the available data by simultaneously determining object function and coil maps from all measured data in a single step.

Parallel imaging as a non-linear inversion problem

The object function (spin density) $r(x)$ and coil sensitivities $c(x)$ relate to the measured signal $y_i(k)$ via the equation $y_i(k) = c_i(k) * r(k) = F\{c_i(x) \cdot r(x)\}$. In other words, the unknown Fourier transform of the spin density is convoluted with the unknown Fourier transform of the coil sensitivity and only a subsampled part of the data is acquired. This equation shows why a determination of the coil sensitivity from the k-space center is problematic. While it holds true that the coil sensitivity maps can be approximated by very few low-order Fourier coefficients, the object function has significant energy even in higher Fourier coefficients. It therefore shifts information of the coil sensitivity map from the center to outer parts of k-space. In order to remove the object function, many autocalibrating methods divide the low-resolution coil images by a low-resolution body coil image or use relative coil sensitivities (GRAPPA) calculated directly from the center of k-space. Information from the undersampled outer parts of k-space is not used. Here, we attempt to directly solve the above equation for the coil sensitivities and the object function at the same time using all available (measured) data. The strategy faces the problem that there are many possible solutions: for each solution every factorization of the product of coil maps and object function into another pair of functions is again a solution. To select a reasonable solution, *a priori* knowledge has to be used. For this purpose it is assumed that the coil sensitivity maps are smooth and the object is homogeneous on a large scale.

Method

We consider the non-linear operator $F : (c,r) \rightarrow y$ and solve the non-linear equation $F(x) = y$ with a Newton method. Given an initial estimate x_n we calculate the next estimate $x_{n+1} = x_n + dx$ by solving the linearized equation $DF(x_n)dx + F(x_n) = y$ for the update dx. Because this equation is a very large and ill-conditioned linear system, we apply the conjugate gradient algorithm in combination with a regularization method to solve it. There exists a whole family of Newton-type regularization methods we can choose from: Extending the normal equation by an explicit regularization term yields the well known Levenberg-Marquardt algorithm which was used in this work. The smoothness of the coil sensitivity maps is enforced by using an appropriate norm (a Sobolev norm). As an example we provide experimental images (2D FLASH, 256×256 matrix) where the number of reference lines is so much reduced that common reconstruction methods start to fail. To demonstrate the performance of the new method we apply it to the limiting case of a reduction factor of 4 with just 4 receiver channels and a low number of reference lines of only 16. The image reconstructed by the GRAPPA algorithm shows remaining ghosting artifacts (Fig. 1). The mSENSE reconstruction is affected by some residual ghosting too and shows a large amount of g-factor noise (Fig. 2). The two images on the right-hand are reconstructed by solving the non-linear equation with the Levenberg-Marquardt algorithm. Figure 3 still reveals some moderate noise amplification but completely removes all undersampling artifacts. In Figure 4 the values of the image were additionally constrained to be real which suppresses noise effectively. All measurements were performed on a Siemens Tim Trio using a 12-channel head coil in CP-mode (using only the four main modes) at 2.9 T.

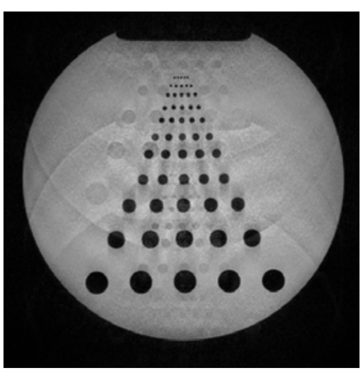

Fig. 1: GRAPPA

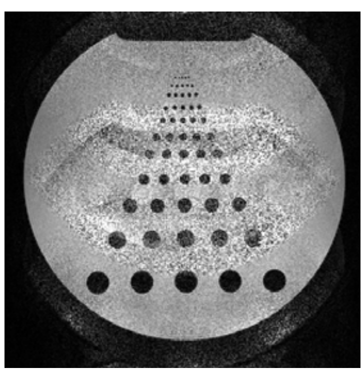

Fig. 2: mSENSE

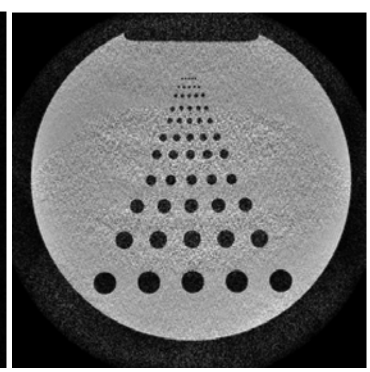

Fig. 3: Non-linear inversion

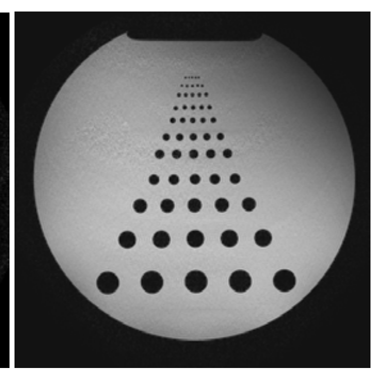

Fig. 4: Non-linear inversion with real value constraint

Conclusion

We propose to view parallel imaging as a non-linear inversion problem and to solve for the unknown coil sensitivity maps and the unknown spin density at the same time. This strategy uses all available data to estimate both the coil sensitivities and object function. It seems to be superior to the commonly used two-step approaches which first estimate the coil sensitivities from only a part of the data and then solve a linear equation where the coil sensitivities remain fixed. Although the extension of the reconstruction process to a non-linear system of equations seems to represent a large complication, it opens a number of advantageous possibilities. For example, the non-linear inversion strategy offers many new ways to incorporate more *a priori* knowledge or to model the measurement process in the system equation more accurately (e.g., by incorporating relaxation maps). In fact, the move from linear to non-linear methods seems to be the next logical step in MR image reconstruction.