

# Undersampled Radial MRI with Multiple Coils. Iterative Image Reconstruction Using a Total Variation Constraint

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**Introduction:** Radial acquisition techniques are recently gaining strong interest due to unique beneficial properties like low sensitivity to motion. The advantages of radial trajectories are accompanied by an increased complexity of the image reconstruction which is conventionally done using projection reconstruction or regridding. In the more frequently used regridding approach the data is interpolated from the sampled spokes onto a grid and subsequently processed by Fourier transformation. This technique provides accurate image quality if a high number of spokes is sampled which, according to the literature, requires a prolonged acquisition of about 57% relative to that of a fully sampled Fourier image (1). If the number of spokes is reduced far below the recommended value, streaking artifacts appear in the image because the gaps in-between the measured spokes remain zero in the regridding reconstruction. Hence, the regridding approach does not give an accurate estimate of the object's Fourier transform in case of a low number of spokes. Interestingly, however, much object information remains visible even when using a low number of spokes like 24. This finding motivated us to design a reconstruction approach that takes the undersampling into account and allows reducing the spokes needed for a proper reconstruction of the object.

**Methods:** In the proposed method the image reconstruction is treated as inverse problem. If an image  $x$  is given, the corresponding spoke data  $y$  can be calculated basically by Fourier transformation of the image and subsequent interpolation onto the spokes by convolution with a Kaiser-Bessel kernel. These linear operations can be combined into a matrix  $A$  leading to the equation  $Ax = y$  [1]. The objective of the reconstruction problem is to obtain an image  $x$  from given measured data  $y$ . Instead of inverting Eq. 1, we use an iterative approach to find an image  $x$  that fits best to the measured data  $y$ . This can be done by minimizing  $\Phi(x) = \frac{1}{2} \|Ax - y\|_2^2$  [2]. However, Eq. 2 only measures the goodness of the image estimate at the acquired spoke

positions. In order to reduce undersampling artifacts it is necessary to extend Eq. 2 by penalty functions that rate the plausibility of the reconstructed object. This drives the algorithm to find a solution complying with prior object knowledge out of all solutions matching at the measured positions. The extended functional takes the form  $\Phi(x) = \frac{1}{2} \|Ax - y\|_2^2 + \sum \lambda_i R_i(x)$  [3] where  $R_i$  are the penalty functions and  $\lambda_i$  are weighting factors. A solution to this

equation can be found using a non-linear conjugate gradient method (2). The proposed approach uses this concept in a two-step mechanism: a coil profile step and a final image estimation step. In the first step, each coil is treated independently to render a complex valued image for each channel. As it is known that coil profiles are smooth functions, edges are penalized strongly by quadratically constraining the derivative at every pixel position. Further, all intensity outside a circular FOV can usually be considered as artifactual and, thus, is also penalized. After finishing the iterations for all coils a sum-of-square image is calculated. A division by the single channel images gives the respective coil profiles. For the second reconstruction step, the data from all coils is stacked into the data vector  $y$  and the matrix  $A$  is extended by a multiplication with the corresponding coil profile before performing the Fourier transformation for every channel. By combining data available from all coils into the data vector, the second step renders a real valued image that complies with the observations from all coils. Again, all image intensity outside the circular FOV is penalized during the iterations. As the streaking artifacts form a texture-like pattern in the images, which can be clearly seen in Fig. 1, the total variation of the image estimate is used as a second penalty function. The total variation is given by the sum of the modulus of the derivative at every pixel. The underlying assumption is that the object consists of piecewise constant areas which applies quite well to medical tomographic images. Therefore, the true object representation out of all solutions matching the measured data should be given by the one with the lowest total variation. This concept is well known in the context of image denoising due to its edge preserving properties (3). Further, as the second reconstruction step renders a real valued image non-negativity can be used as a third constraint by quadratically penalizing negative values. This prevents the algorithm from inserting undesired negative fill values to better match the measured data. As a first proof-of-principle application of the method we used a radial 2D spin-echo sequence to avoid complications from off-resonance effects. The measurements were performed using a Siemens Tim Trio system at 2.9T with a 12-channel head coil in triple mode. All images were acquired with a base resolution of 256 pixels covering a 230 mm FOV (bandwidth 180 Hz/pixel). A readout oversampling factor of two has been used as well as an isotropic gradient delay correction.

**Results:** Fig. 1 shows phantom reconstructions from 48 and only 24 spokes using the regridding approach (upper row in all figures) and the proposed method (lower row). Accordingly, Fig. 2 shows reconstructions of the human brain in vivo with T2 contrast. It is clearly visible that the regridding reconstructions suffer from streaking artifacts that increase with lower number of spokes. The proposed method is able to reconstruct the objects without or at least strongly reduced streaking artifacts. Further, the use of the total variation penalty leads to a certain degree of image denoising. The improvement in image quality is even better visible in Fig. 3 showing magnifications from Fig. 2 by a factor of three.

**Conclusion:** This work presents a new approach for the iterative reconstruction from undersampled radial MRI acquisitions, which is able to handle data from multiple coils and allows to incorporate prior object information by introducing suitable penalties. In particular, constraining the total variation of the reconstructions led to an effective reduction of streaking artifacts that normally limit the application of radial undersampling strategies. This enables to obtain images from only a very limited number of spokes with significantly improved quality compared to conventional radial reconstructions.

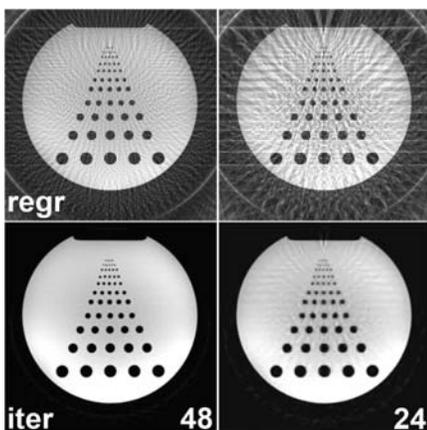


Fig. 1 Phantom data (TR/TE=4000/11ms)

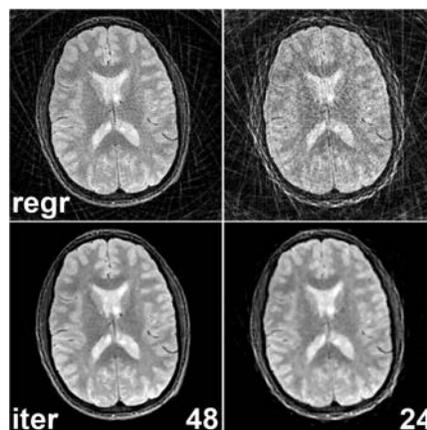


Fig. 2 Human brain data (TR/TE=2500/50ms)

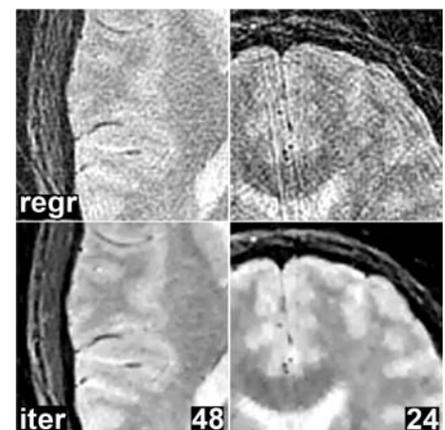


Fig. 3 Magnified sections from Fig. 2

**References:** 1. Bernstein, M.A. et al, Handbook of MRI Pulse Seq., Elsevier Acad. Press, 2004. 2. Hager, W. et al, SIAM J Opt,16:170-192,2005. 3. Rudin, L.I et al, Physica D,60:259-268,1992.