A Sparse Matrix Formalism for Non-Rigid Motion Correction

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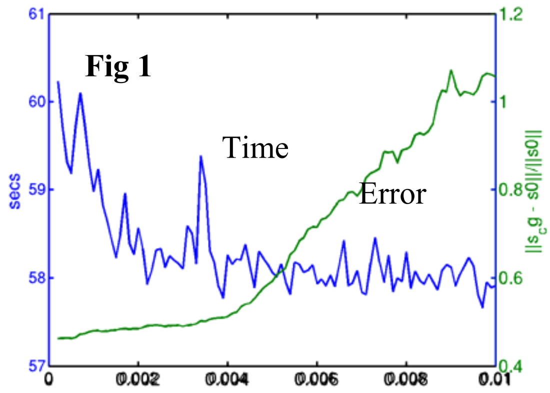
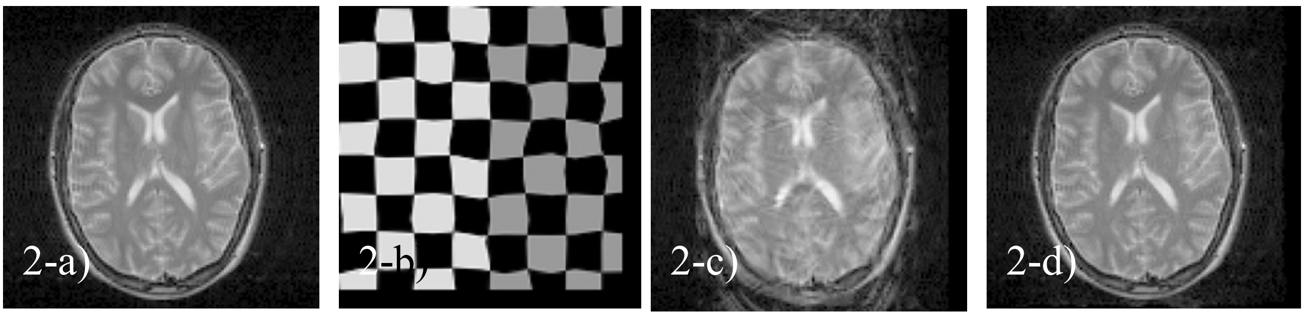
Introduction. Motion correction in MR has gone from being limited to 1D, or at best rigid motions to much more complicated non-rigid corrections [1]. This is achieved by building an *implicit* linear system whose unknown is the non-ghosted image, and whose known is the ghosted one. This, however, requires knowledge of the inverse of the spatial transform. Here we discuss a method to address this issue by constructing *explicitly* a sparse system for the unknown image, whose right hand-side is the motion-ghosted image, and whose sparse matrix can be identified with the collection of space-variant Point Spread Functions (PSF). We make the assumption that motions occur between shots, in a multishot context, where each shot is a set of regularly spaced lines. The motion is assumed measured in some way, which is possible ([3,4,5]).

Theory and Methods. Rotation in image space corresponds to rotation in k-space. This statement cannot be generalized to nonlinear motions, and is not sufficient to deal with motion happening during acquisition. If this motion can be modeled, it has been shown that it is possible to solve the ghosting equation, thus correct even nonlinear motion [1]. The main equation states that the motion ghosted image can be written as $\boldsymbol{s}=(\sum_t \boldsymbol{a}_t\boldsymbol{u}_t)\boldsymbol{s}_0 =: \boldsymbol{\gamma}\boldsymbol{s}_0$ where $\boldsymbol{a}_t$ is the t-th *aliasing* matrix representing the aliasing corresponding to the sub-sampling at time t, and $\boldsymbol{u}_t$ is a linear operator representing motion on the space of images. Thus, the operator $\boldsymbol{\gamma}$ is a linear operator and one can in theory solve for $\boldsymbol{s}_0$.This, however, assumes that the motion is known, and that we can perform the multiplication in reasonable times. Note that as γ operates on the space of images (*e.g.* $N=256^D$-dim. in D dims), such a matrix would require a huge N^2 storage, which is unrealistic. This was solved in [1] by --implementing the product with $\boldsymbol{\gamma}$ with the help of standard FFT and sub-sampling to get a matrix-vector product function which could be fed into a Conjugate Gradient of Normal equation (CGNE) algorithm (LSQR) --optimizing over all possible motions in a motion model. This, however, raises a difficulty. Because the CGNE requires multiplication with the *transpose*, one needs to transpose the operation corresponding to spatial transforms, which amounts to *inverting* the spatial transform. This limits considerably the space of potential non-rigid transforms, as for most of them the inverse is unknown. Here we solve this issue by, instead of using a matrix-vector form, building explicitly a sparse NxN matrix $\boldsymbol{\gamma}_{sp}$, whose columns can be interpreted as the space-variant PSFs for the N pixels. The sparseness is controlled by a threshold parameter, *i.e.*, a matrix element is kept if its magnitude is higher than the threshold. In this way, the transposition operation is just a standard matrix operation, and non-rigid motions, whose inverse is unknown, such as typical spline-based deformation, can be used practically. Note that $\boldsymbol{\gamma}_{sp}$ also depends on the interpolation method, which allows further trade-offs between speed, (sparseness) and error.



Results. The graphs in **Fig. 1** show the computation times and relative errors for a simulated spline deformation (computed from a 32x32 image), for sparseness threshold from 0.001 to 0.01. **Figure 2** shows a higher resolution simulation, where a 128x128 image $\boldsymbol{s}_0$ (**Fig. 2-a**) is subjected to a shot dependent spline deformation (**Fig. 2-b** shows deformation at shot 2), here in 8 shots, producing the ghosted image $\boldsymbol{s}$ (**Fig. 2-c**). The sparse matrix $\boldsymbol{\gamma}_{sp}$ is constructed for a threshold of 0.0001, and a corrected image is constructed by solving the equation $\boldsymbol{s} = \boldsymbol{\gamma}_{sp}\boldsymbol{s}_0$ (**Fig. 2-d**) (total computation time ~90mins**).**



Conclusion-Discussion. We have shown that ghosts due to general types of (known) motion, whose inverse is not even known, can be fully corrected, in a Cartesian multi-shot context. This could be used by for example using information from navigators or statistical deformation models [3,4,5] to find what motion occurred, and to apply this correction directly to the ghosted images. The sparse matrix point of view puts this work in the context of 'space-variant PSF' [2]. **Figure 1** shows what the associated cost of the sparse matrix version is, and how the errors grow with sparseness threshold (a good choice of threshold will depend on image resolution, magnitude of data, and other factors such as interpolation). There is a limiting value above which the errors become too large, and not much time is gained, but this method allows extremely general types of motion, which would compensate for such limitations.

References: [1] Batchelor et al, *MRM*, **54**, 1273-1280 (2005). [2] Nagy and O'Leary, *SIAM J. Sci. Comp*. **19**, 1063-1082 (1998). [3] McLeish et al, *IEEE TMI,* **21,** 1142-1150, (2002). [4] Irrarazaval et al, MRM, **54,** 1207-1215, (2005). [5] McLeish et al, *MRM*, **53**, 1127-1135, (2004).

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