Understanding the GRAPPA Paradox

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Introduction GRAPPA [1] uses convolution kernels in *k*-space to synthesize phase-encode lines that have not been acquired. The autocalibration procedure used by GRAPPA can also be employed in an image space reconstruction scheme: by Fourier transforming the convolution kernels, “unfold” images can be computed. While these unfold images are used in the same manner as unfold images in SENSE [2] reconstructions, where the unfold images are multiplied by the corresponding aliased coil images and summed to reconstruct an unaliased image, the unfold images themselves are distinctly different between these methods. Since GRAPPA uses a convolution kernel of very limited extent in *k*-space, the corresponding unfold images do not contain any high spatial frequency components. Yet GRAPPA performs well even when the coil sensitivities contain high spatial frequency components. In contrast, self-calibrating SENSE [3] does not restrict the unfold image to low spatial frequencies, but self-calibrating SENSE does not perform as well when the coil sensitivities contain high spatial frequencies.

It is this paradox we explore in this work: how can GRAPPA’s low-frequency kernel accurately reconstruct images encoded with coil sensitivities that have high-frequency components? This paradox is especially relevant when the prescribed FOV is smaller than the object, in which case the reconstruction problem can be modeled using aliased coil sensitivities that have high frequency edges [4]. In this work, we compare the unfold images generated by GRAPPA to those used by self-calibrating SENSE to show that they are not equivalent—a realization we hope will elucidate the practical differences between these methods.

Theory Since the number of coils used in a parallel imaging experiment is typically greater than the acceleration factor, the reconstruction problem is overdetermined. Thus while there is a single set of coil sensitivities used to encode an object, there are an infinite number of unfold images that will reconstruct the correct image. Self-calibrating SENSE assumes that the estimated coil sensitivities are correct and finds the unfold images that maximize the signal-to-noise ratio. Any inaccuracies in self-calibrating SENSE reconstructions are due to inaccuracies in the coil sensitivity estimation. When the coil sensitivities have high spatial-frequency components, the self-calibration procedure is not able to accurately estimate the coil sensitivities. In contrast, the autocalibration procedure used by GRAPPA finds unfold images containing only low spatial-frequency components that minimize the reconstruction error. High frequency components in the coil sensitivities do not preclude the existence of low frequency unfold images, especially when the problem is highly overdetermined. Because GRAPPA does not suffer from errors due to inaccuracies in estimated coil sensitivities, it does not have difficulty with high spatial-frequency components in the coil sensitivities. Nor does it have difficulty with high spatial-frequency components in the object, such as the sharp intensity jump at the edges of bright subcutaneous fat.

Methods Simulations were performed on a numerical Shepp-Logan head phantom in Matlab (MathWorks, Natick, MA), using an 8-element coil arrangement with the same parameters as in [4], a 256x256 matrix, an acceleration of 2 and 12 autocalibrating (ACS) lines. Next, volunteers were imaged at 3.0T (GE Healthcare, Waukesha, WI) with an 8-channel head coil (MRI Devices, Waukesha, WI). A spoiled gradient echo sequence using the same trajectory as in the simulation was used to acquire an accelerated dataset with the following parameters: TE/TR= 3.4/150ms, FOV=22x16 cm, 256x180, α=30°, BW=±31kHz, slthick=5mm. The image FOV was deliberately prescribed to be slightly smaller than the object to introduce high spatial frequency content into the coil sensitivities. For both experiments, coil-by-coil unfold images were calculated using both self-calibrating SENSE and the GRAPPA autocalibration procedure. A Kaiser window (β=4) was applied to the extracted central k-space data in the self-calibrating SENSE coil sensitivity estimation step and the 2D GRAPPA kernel widths were 5 and 4 in the kx and ky directions, respectively. Image reconstruction was performed offline in Matlab using the calculated unfold images.

Results Figure 1 shows the results of the numerical phantom experiment. Even though the FOV fully encompasses the object, the self-calibrating SENSE image shows significant residual aliasing artifacts due to its inability to fit the high-frequency edge of the phantom. GRAPPA, however, reconstructs the same data with no visible artifact. Figure 2 compares the *in vivo* data reconstruction results. Since the prescribed FOV was slightly smaller than the object in the phase-encoding direction, residual aliasing artifact is visible in the center of the self-calibrating SENSE reconstruction (c,d) due to the inability of the self-calibration to accurately estimate the sharp discontinuities in the aliased coil sensitivities. Note that the unfold images in (b) and (f) look nothing alike. Even though GRAPPA’s unfold image contains no high frequency information, it still performs well when reconstructing an image encoded with high spatial frequency coil sensitivities.

Discussion Although GRAPPA determines its kernel weights with a limited region of support in *k*-space, this does not imply an assumption of slowly varying coil sensitivity functions in image space. As these results show, low-resolution unfold images can reconstruct images encoded with high-resolution coil sensitivities. As the number of receiver coils increase, the reconstruction will become even more overdetermined and will further lend itself to GRAPPA, whereas more coils will do nothing to improve the coil sensitivity estimation step of self-calibrating SENSE. This work underscores the fact that autocalibrating methods such as GRAPPA and self-calibrating SENSE represent fundamentally different approaches to solving the reconstruction problem.

References [1] Griswold MA et al. MRM 47:1202-1210. [2] Pruessman et al. MRM 42:952-962, 1999. [3] McKenzie et al. MRM 47:529-538, 2002. [4] Griswold MA et al. MRM 52:1118-1126, 2004.

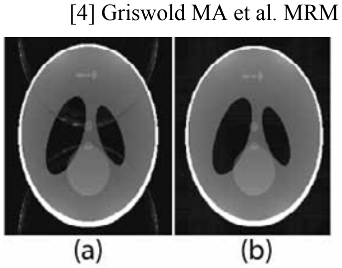

Figure 1. (a) Numerical phantom data reconstructed with self-calibrating SENSE. Note the marked residual aliasing of high-frequency edges. (b) Same data reconstructed using autocalibration (GRAPPA).

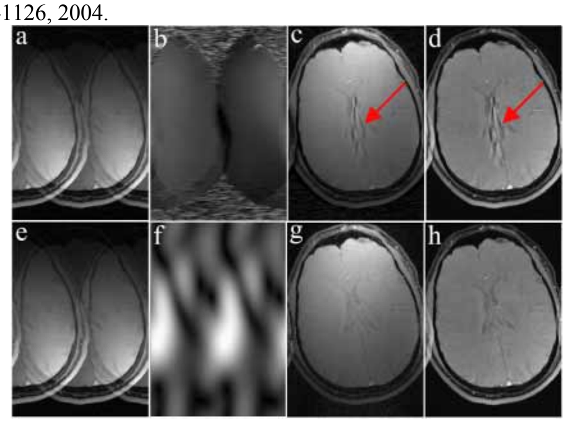

Figure 2. Top row: self-calibrating SENSE. Bottom row: GRAPPA (a,e) Aliased head image from coil 4. (b,f) Unfold image applied to coil 4 to reconstruct coil 1. Note that the unfold images produced by the two methods show little resemblance. (c,g) Reconstructed coil 1 image. Note the aliasing artifact at the center of the image in (c). (d,h) Reconstructed sum-of-squares image. Note the aliasing artifact at the center of the image in (d).