From ADC to Probability Profiles: The Diffusion Orientation Transform

E. Ozarslan[1,2], T. M. Shepherd[2], B. C. Vemuri[2], S. J. Blackband[2], T. H. Mareci[2]

[1]National Institutes of Health, Bethesda, MD, United States, [2]University of Florida, Gainesville, FL, United States

INTRODUCTION

Diffusion-weighted MRI provides a non-invasive means to map the neural connections between functionally connected regions of the central nervous system. High angular resolution diffusion imaging (HARDI) [1] has the potential to capture the orientational structure even in regions with complicated architecture from relatively limited number of scans at lower diffusion-weightings. However, an important challenge is to reliably map the signal (hence the apparent diffusivity) profiles into estimates of the water displacement probabilities. The *Diffusion Orientation Transform* achieves this by establishing a direct link between the diffusivity and probability profiles.

THE TRANSFORM

The DOT utilizes the Fourier relationship between the diffusion-weighted MR signal attenuations and the particle displacement probabilities. The Fourier transform is expressed in spherical coordinates and the radial part of the integral is evaluated analytically. Probabilities of water displacements at a particular distance away from the origin can be efficiently computed using either one of two reconstruction methods. The *parametric reconstruction* yields the probability values in terms of spherical harmonics. A schematic description of the DOT method with parametric reconstruction is provided below:

$$E(\mathbf{u}) \xrightarrow{\text{Eq.(1)}} D(\mathbf{u}) \xrightarrow{\cdots} I_l(\mathbf{u}) \xrightarrow{\times(-1)^{l/2}\times\mathsf{SHT}_l} p_{lm} \xrightarrow{\text{LS}} P(R_0\mathbf{r})$$

Alternatively, the *nonparametric reconstruction* makes it possible to directly estimate the probability profiles as outlined below:

$$E(\mathbf{u}) \xrightarrow{\text{Eq.(1)}} D(\mathbf{u}) \xrightarrow{\cdots} I_l(\mathbf{u}) \xrightarrow{\text{Eq.(2)}} P(R_0\mathbf{r})$$

In the above schematic descriptions, SHT_l and LS stand for the l-th order spherical harmonic transform and Laplace series respectively. $E(\mathbf{u})$ and $D(\mathbf{u})$ are respectively the angular signal attenuation and apparent diffusivity profiles, whereas $P(R_0\mathbf{r})$ is the probability of water molecules to move a distance R_0 along the direction $\mathbf{r}$. The operation that transforms a diffusivity profile into the intermediate function $I_l(\mathbf{u})$ is derived from the radial part of the Fourier transform. Eqs.(1) and (2) are given by

$$E(\mathbf{u}) = e^{-bD(\mathbf{u})} \qquad (1) \qquad \text{, and} \qquad P(R_0\mathbf{r}) = \sum_{l=0}^{\infty} \int d\mathbf{u}\, (-1)^{l/2} \frac{2l+1}{4\pi} P_l(\mathbf{u}\cdot\mathbf{r})\, I_l(\mathbf{u}) \qquad (2)$$

The computations of the p_{lm} coefficients take less than a minute for most three-dimensional data sets. Furthermore, a simple implementation of the transform is possible through a matrix formulation of the above scheme.

RESULTS

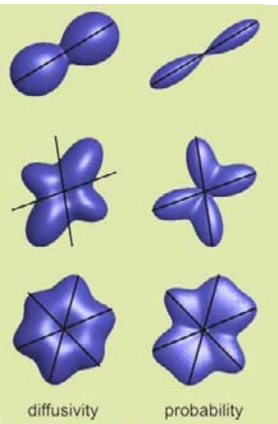


Figure 1 shows the simulations of 1-, 2- and 3-fiber voxels. The simulated experiments consisted of 81-direction diffusion-attenuated signal profiles at a b-value of 1500s/mm^2. As demonstrated on the left column, the ADC profiles do not necessarily yield correct fiber orientations. However, the peaks of the probability values computed using the DOT coincide with the true fiber orientations as depicted by the solid black lines.

Table 1 shows the angular deviations (in degrees) of the computed fiber orientations from the true fiber directions. The second column presents the deviation angles when no noise was added. Columns 3-6 show the mean and standard deviation of the deviation angles when Gaussian noise of standard deviation σ was added to the real and complex components of the signal. The computations were repeated 100 times for each noise level.

	$\psi\,(\sigma=0)$	$\psi\,(\sigma=0.02)$	$\psi\,(\sigma=0.04)$	$\psi\,(\sigma=0.06)$	$\psi\,(\sigma=0.08)$
1 fiber	$\{0.364\}$	0.77 ± 0.42	1.44 ± 0.79	2.20 ± 1.09	3.08 ± 1.66
2 fibers	$\{1.43, 0.80\}$	2.33 ± 1.10	3.66 ± 2.01	6.00 ± 5.57	8.07 ± 7.92
3 fibers	$\{2.87, 0.60, 4.57\}$	5.81 ± 5.84	11.5 ± 10.1	14.7 ± 10.3	17.6 ± 11.9

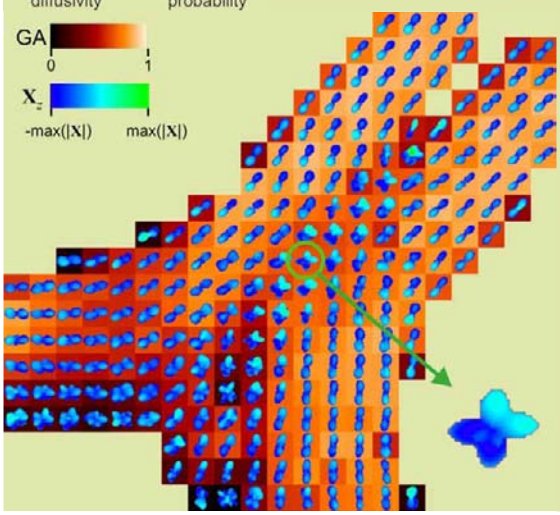


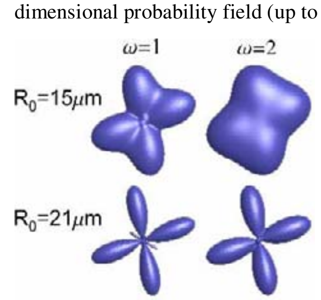
Figure 2 depicts the probability maps calculated from a rat optic chiasm data set overlaid on axially-oriented generalized anisotropy (GA) maps. The decussations of myelinated axons from the two optic nerves at the center of the optic chiasm are readily apparent using the DOT method. A total of 46 diffusion-weighted images were acquired at 14.1T with a b-value of 1250s/mm^2. Diffusion gradient separation (Δ) was 12.4ms, and the pulse duration (δ) was 1.2ms. Resolution of the images was 33.6x33.6x200μm^3. The images were voxel averaged to 67.2x67.2x200μm^3 resolution.

A SIMPLE SMOOTHING STRATEGY FOR THE DOT

When the HARDI experiment is performed such that only a spherical shell in q-space is sampled in addition to the origin, then multiplying the signal value at the origin by some constant ω results in an isotropic smoothing of the probability field. This is because the ratio of the new signal attenuation values to the original ones is given by

$$\frac{E'(\mathbf{q})}{E(\mathbf{q})} = \omega^{1-q^2/q_1^2}$$

An interesting choice is $\omega=e$. In this case, since the attenuation values, which reside in the frequency space for the probability, are multiplied with a Gaussian function, this choice results in an exact Gaussian convolution of the three-dimensional probability field (up to a multiplication by e).



This approach may be useful in DOT reconstruction from very noisy signal. As shown on the left, simulations suggest that when ω is greater than 1, the probability profiles get smoother, and spurious peaks disappear at large values of R_0.

Reference: [1] D. S. Tuch et al., *Proc Intl Soc Magn Reson Med*, p.321, (1999).