The effects of magnetic field inhomogeneity on FLASH-based T_1 measurements

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Introduction The use of multiple FLASH experiments with different flip angles and/or repetition times has been proposed as an efficient, fast, low SAR method for measuring T_1 in vivo [1]. Recent work [2, 3] used two measurements with different flip angles and a single short *TR* to measure T_1 in vivo at high resolution. In this work we present a quantitative analysis of the effects of B_1 inhomogeneity on FLASH-based T_1 measurements. We also investigate the use of FLASH sequences to measure the B_1 inhomogeneity itself, as proposed by [4].

It is well known that at infinite *TR* the FLASH signal is independent of T_1 and two measurements at different flip angles suffice to estimate spin density and B_1 exactly. We show that at long but finite *TR* the effect of T_1 is to introduce a small and systematic error in the estimates of B_1 and spin density. We show analytically that at short *TR* there exists a continuum of solutions that fits the data equally well. Through numerical experiments we demonstrate that the simultaneous estimation of the physical parameters T_1, spin density and B_1 from a set of finite-*TR* FLASH data is numerically ill-posed, irrespective of the number of flip angles and *TR*s, short or long.

Theory and results of numerical experiments

The FLASH signal equation is given by $S(f\alpha) = M_0(1-\exp(-TR/T_1))\sin f\alpha / (1-\exp(-TR/T_1)\cdot\cos f\alpha)$ where α is the nominal flip angle and M_0 is the spin density, and the multiplicative factor f is a measure of B_1 inhomogeneity that relates the nominal flip angle to the true flip angle at a particular point at the sample.

At infinite *TR* the FLASH equation no longer depends on T_1, $S(f\alpha) = M_0 \sin f\alpha$. Two measurements at nominal angles α_1 and α_2 suffice to calculate the field bias f and spin density M_0.

At long but finite *TR*, the FLASH equation is approximated as

$$S(f\alpha) = M_0(1-\exp(-TR/T_1))\sin f\alpha + M_0\exp(-TR/T_1)\sin f\alpha\cos f\alpha = M_0^{eff}\sin f\alpha + \text{(effective error term)}$$

For example, estimating the B_1 non-uniformity by an "infinite-*TR*" experiment where TR/T_1=3 results in B_1 estimation error on the order of 5% and M_0 estimation error also on the order of 5%.

At very short *TR* and small flip angles the FLASH equation is approximately

$$S(f\alpha) \cong M_0 f\alpha / \left(1 + \left(f^2 T_1\right)\alpha^2/2TR\right)$$

We observe that the sets of parameters (M_0, T_1, f) and (fM_0, f^2T_1, 1), related by a continuous scaling transformation, produce exactly the same signal. This also shows that the systematic error in T_1 is quadratic in B_1 and the systematic error in M_0 is linear in B_1.

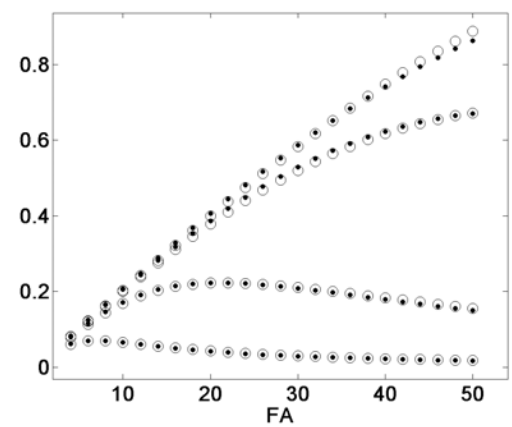

Figure 1: FLASH data were synthesized using FA=4°, 6°, 8° … 50°, *TR* = 10ms, 100ms, 1000ms, 5000ms, T_1=1000ms, M_0=1, and f=1.2 (circles). A nonlinear least-squares fit was performed to solve for T_1 and M_0 with f held constant at f=1.0. The fit yields T_1=1375ms and M_0=1.17. The signal calculated with the estimated parameters (1375ms, 1.17, 1) (dots) is superimposed on the original data (circles). The total sum of residuals square, $\chi^2=3.3\times10^{-3}$. The maximum signal divided by $(\chi^2)^{1/2}$, approximately 15, is a back-of-the-envelope estimate of the minimum SNR necessary to distinguish the difference between the two sets of parameters in this experiment.

Three more sets of FLASH data were synthesized using the same parameters with f =0.6, 0.8, and 1.4. The procedure above was applied resulting in estimated T_1 and M_0 shown in **Table 1**.

	T_1(ms)	M_0(a.u.)	χ^2	max(S)/$(\chi^2)^{1/2}$
f=0.4	183	0.42	2.3×10^{-3}	7.0
f=0.8	665	0.82	1.4×10^{-3}	16.9
f=1.2	1375	1.17	3.3×10^{-3}	15.0
f=1.6	2300	1.47	4.9×10^{-2}	4.4

The scaling relationship between parameter sets (T_1, M_0, f) and ($T_1(f)$, $M_0(f)$, 1) persists over several decades of *TR*. Due to this scaling relationship, B_1 cannot be intrinsically estimated together with M_0 and T_1 using FLASH, but must be measured independently.

Summary Multiple FLASH experiments with different flip angles and/or repetition times have been proposed as an efficient, fast, low SAR method for measuring T_1 in vivo. Using analytical arguments and numerical experiments we show that the simultaneous estimation of the physical parameters T_1, spin density and B_1 from finite-*TR* FLASH data is ill-posed, irrespective of the number of flip angles and *TR*s, short or long.

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References

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