A Simple Relationship for High Efficiency-Gradient Uniformity Trade-offs in Multi-Layer Asymmetric Gradient Coils

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Introduction: This study details the design trade-offs available for asymmetric gradients and particularly investigates both scaling laws and appropriate relaxation factors useful in the design process. Recently a linear function interrelating coil diameter and DSV with the shortest length high performance symmetric gradient coils has been presented [1]. It is well-known that relaxing the magnetic field quality is one of the ways to obtain gradient coils with high figure of merit $M=(\eta^2\rho_1^5)/L$. It is not clear, however, how this relaxation factor is related with DSV size, coil length and radius to produce maximal M. In this work, we have studied the influence of DSV, coil length and radius, relative axial offset position of DSV and target gradient field uniformity over the figure of merit in multi-layer asymmetric transverse gradient coils. A simple linear function that defines the optimal coil length to produce a maximum figure of merit given a DSV size, coil radius, axial offset position and introduced uniformity error is obtained.

Method: The method assumes N layers of the current density $\mathbf{J}(\rho,\phi,z)$ flowing in concentric cylindrical surfaces of radii ρ_n. The $\mathbf{J}(\rho,\phi,z)$ is confined in the interval $(0\leq z\leq L_n)$. $\mathbf{J}(\rho,\phi,z)$ in each layer is expressed as a sum of Q orthonormal functions multiplied by the amplitudes λ_{nq} [2]. In our approach the magnetic field $B_z(\rho,\phi,z)$ and its gradient along x-coordinates are derived from the Biot-Savart law. The multi-layer asymmetric gradient coil design is stated as a quadratic optimization problem with linear field constraints:

$$\mathrm{Min:}\sum_{n=1}^{N}\sum_{q=1}^{Q}\sum_{n'=1}^{N}\sum_{q'=1}^{Q}\left(\lambda_{nq}\lambda_{n'q'}C_{nq}C_{n'q'}\right) \qquad (a)$$

Linear Constraints:

$$\sum_{n=q}^{Q}\sum_{n=1}^{N}G_{x_{qnt}}\cdot\lambda_{nq}\leq G_0(1+\varepsilon)\quad t=1,\ldots,\mathrm{T}, \qquad (b)$$

$$-\sum_{q=1}^{Q}\sum_{n=1}^{N}G_{x_{qnt}}\cdot\lambda_{nq}\leq -G_0(1-\varepsilon)\quad t=1,\ldots,\mathrm{T}, \qquad (b)$$

$$\left|\sum_{q=1}^{Q}\sum_{n=1}^{N}\left(B_{z_{xqnt}}\cdot\lambda_{nq}\right)\right|\leq B_{zt,\mathrm{shield}}\quad t=\mathrm{T}+1,\ldots,\mathrm{T}+\mathrm{P}, \qquad (c)$$

$$\left|\sum_{q=1}^{Q}\sum_{n=1}^{N}\left(T_{y_{qnt}}\cdot\lambda_{nq}\right)\right|\leq T_{y0}\quad t=\mathrm{T}+\mathrm{P}+1,\ldots,\mathrm{K}, \qquad (d)$$

$$\left|\sum_{q=1}^{Q}\sum_{n=1}^{N}\left(F_{x_{qnt}}\cdot\lambda_{nq}\right)\right|\leq F_{x0}\quad t=\mathrm{T}+\mathrm{P}+\mathrm{K}+1,\ldots,\mathrm{U} \qquad (e)$$

The magnetic energy (a) is calculated in the Fourier space [2]. G_o is the target gradient strength in [T/m], G_{xqnt} is gradient contribution at the target point t of the axial mode q oscillating in the cylindrical surface of radius ρ_n; T is the number of target points in the DSV. Instead of balancing the constraints through weighting factors, we have introduced an non-uniformity error ε. This parameter allows a control over desired target gradient uniformity and hence with the desired M trade-off. P and $B_{zt,\mathrm{shield}}$ in (c) are the number of target points and field value in the shielding surface, respectively. T_{y0} and F_{x0} are the target torque and force values in the K and U elemental areas in the layer n. Different from other methods the inequality linear constraints (c,d,e) are not zero to assure practical gradient coil solutions with high M-gradient uniformity trade-off.

Results and Discussions: Thousands of multi-layer asymmetric transverse gradient coils were calculated for different L_1/DSV and fixed values of number of layer (N=1,2,3,4), ε, axial offset position of DSV (z_o) and ρ_1/DSV. In the case N=1, the shielding constraint was not included. Five ε values ranging from 0.1 % to 10% were used. The L_1/DSV and ρ_1/DSV ratios were varied in the range 2 to 5.5 and 1.4 to 2.5, respectively. N radial layers equally spaced between ρ_1 and $1.24\cdot\rho_1$ were located. The axial coil length value was equally distributed between L_1 and $1.2\cdot L_1$ for the N surfaces. A single shielding surface was located at radius $1.5\cdot\rho_1$ and the axial length was set to $1.2\cdot L_1$. Three DSV axial offset values were considered $z_o=(L_1/4.1, L_1/3.6, L_1/3.1)$. Plots of M versus L_1/DSV for fixed values of D_1/DSV, N, ε and z_o show an optimal coil length value that produces maximal M. As expected the M value increases almost 1.5 times when the ε value is increased 2 times. When N ($N\geq 2$) value is increased with fixed values of L_1/DSV, D_1/DSV, N, ε and z_o the M value decreases.

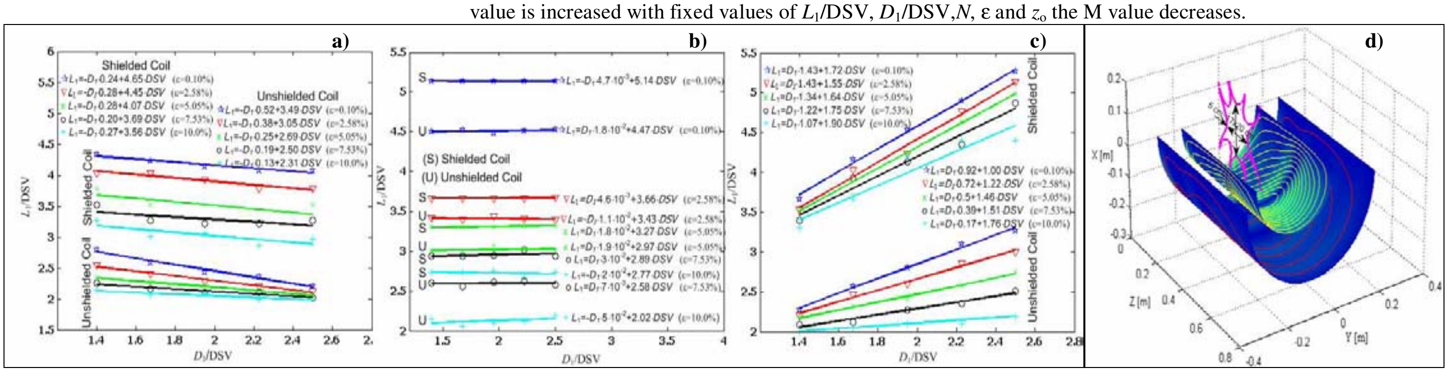

Figure 1. Maximal M coil length (L_1) as function of DSV, D_1 ($D_1=2\rho_1$) and ε for different z_o (**c**)). Coil pattern and maximal pressure over the coil support **d**.

This effect is critical when L_1/DSV≤1. However, in our case when L_1/DSV≥2 the change of M with N can be assumed to be constant. For fixed L_1/DSV, D_1/DSV, N and ε when z_o approaches from the coil end to coil center the M value increases. Coils with DSV>$0.5\rho_1$, $z_o<L_1/4.1$ and ε<5% produce un-reliable current patterns. For fixed ε and z_o the coordinates (D_1/DSV, L_1/DSV) where M is maximum is plotted and fitted based on linear regression producing a relationship for the optimal coil length that produces maximal M as function of D_1, DSV, ε and z_o. Fig. 1, (**a**,**b**,**c**) shows the linear functions for N=1 and $N\geq 2$. Depending on z_o value the linear relationship will present three kinds of dependency with D_1: negative, close to zero and positive slope. Negative means that for $z_o=L_1/4.1$ fixed DSV and ε value the coil length must be reduced when D_1 is increased. Close to zero slope ($z_o=L_1/3.6$) implies that the coil length is determined by the DSV and ε. In other words, the maximal M value appears in the same axial position when the D_1/DSV ratio is varied in the studied range for a fixed ε value. Positive slope indicates that z_o is relatively close to the coil center ($z_o=L_1/3.1$). In this situation negative turns can appear at the end of the coil ($\rho,\phi,0$), minimizing the torque and the Peripheral Nervous Stimulation. For small ε values the coefficient of correlation increases. This implies that target field is over specified and the solution tends to be unique . However, if the ε value is sufficiently relaxed (larger than 7%), the solution space increases and any intermediary solution can be reached within the permissible relaxation target uniformity error ε. Assuming the same geometrical parameter of Tomasi's design [3], applying our rule using ε=7.5% and $z_o=L_1/3.6$, the resulting coil produces an M value equal to $2.92\cdot 10^{-8}$ $\mathrm{T^2m^3H^{-1}A^2}$, which is 1.18 times lager than the M value produced in [3] with similar gradient uniformity in the same DSV. The 5% contour is 5 cm away from the coil edge (Fig.1**d**).

Conclusion: In this work we have studied the relationship between the target field uniformity, DSV size, coil diameter, coil length and the figure of merit M for asymmetric multi-layer transverse gradient coils. A simple linear function that defines the optimal coil length that produces maximum figure of merit given DSV, coil radius, axial offset position and introduced uniformity error has been derived. Applying the method and the linear functions described in the present work, solutions with superior figure of merit – gradient uniformity trade-off are obtained. The benefit of using the target uniformity error rather than the conventional weighting factor is that one can obtain more control over the figure of merit and hence generate a high gradient uniformity to figure of merit trade-off.

References.

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