Accelerating dynamic spiral MRI by algebraic reconstruction from undersampled *k-t* space

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Introduction: The temporal resolution of dynamic MRI can be increased by acquiring only a fraction of *k*-space. The fundamental principle of such approaches is to exploit information redundancy in fully sampled *k-t* space; and the temporal correlation of dynamic images has been considered as useful source of this redundancy. Recent important examples are UNFOLD [1] and *k-t* BLAST [2,3], which economically undersample *k-t* space according to an interleaved sampling function such that aliased signals can be separated from the original signal. They have been used almost exclusively with Cartesian *k*-space trajectories due to the simplicity of describing and correcting for the aliasing. In this abstract, we describe both algebraically and graphically the aliasing process for spiral sampling trajectories, and present strategies for finding solutions to the resulting ill-conditioned linear systems. The proposed method is demonstrated using 2-fold undersampling with spiral *k*-space trajectories in in-vivo experiments.

Theory and Methods: The effects of undersampling are best seen in *xyf* space, the reciprocal of *k-t* space (Fig. 1). Interleaved 2-fold undersampling can be expressed as a 3D convolution between the original object $I(x,y,f)$ and point spread function $S(x,y,f)$, simplified as follows.

$$R = I *_3 S = \sum_{l=0}^{L-1}\left[I^{(l)} *_2 S^{(f-l)}\right] = I^{(f)} *_2 S^{(0)} + I^{(f-L/2)} *_2 S^{(L/2)} \quad (1)$$

where $I^{(l)}$ and $S^{(l)}$ are 2D planes located at $f = l$ out of the whole 3D object I and S. If the 2D convolution matrices associated with $S^{(0)}$ and $S^{(L/2)}$ are $\mathbf{A}_1$ and $\mathbf{A}_2$, the aliasing process can be written as,

$$\begin{bmatrix}\mathbf{A}_1 & \mathbf{A}_2\\ \mathbf{A}_2 & \mathbf{A}_1\end{bmatrix}\begin{bmatrix}\mathbf{x}_l\\ \mathbf{x}_{l+L/2}\end{bmatrix}=\begin{bmatrix}\mathbf{b}_l\\ \mathbf{b}_{l+L/2}\end{bmatrix} \text{ or } \mathbf{A}\mathbf{x}^{(l)} = \mathbf{b}^{(l)}\ (l = 0,\cdots,L/2-1) \quad (2)$$

where $\mathbf{x}_l$ and $\mathbf{b}_l$ are vector versions of $I^{(l)}$ and $R^{(l)}$. Note that only two slices $I^{(l)}$ and $I^{(l+L/2)}$ are coupled into the same linear system.

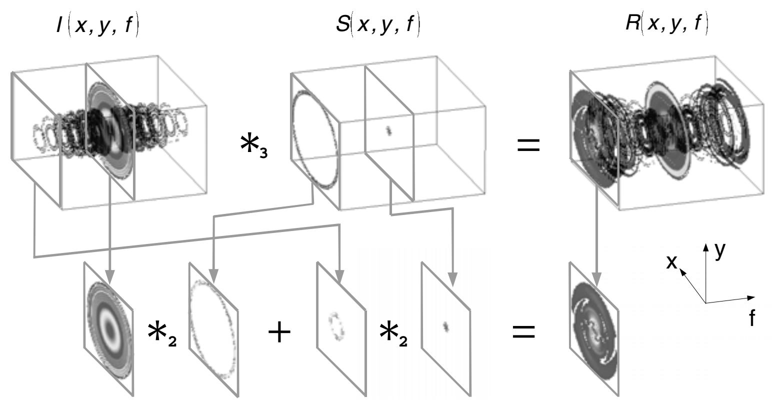

Fig. 1. Illustration of dynamic undersampled spiral imaging using *xyf* space

The system matrix in (2) is numerically rank deficient. To find a solution, zero assumption in the solution can be suggested since its support region is bounded within *xyf* space. From the rank analysis of Cartesian 2-fold undersampling case it can be proven that the mathematical requirement for a unique solution is equivalent to the geometric condition that nonzero portions of $I^{(l)}$ and $I^{(l+L/2)}$ should not overlap each other, which is assumed to carry over into spiral undersampling. That is, if we approximate the sidelobe from spiral undersampling as a simple ring, the summation of the radial extents of support regions in $I^{(l)}$ and $I^{(l+L/2)}$ should be smaller than the nominal FOV. In practice, however, this condition is too demanding and the system matrix after zero assumption tends to be still ill-conditioned. Hence we use Tikhonov filtering which produces a regularized linear system $(\mathbf{A}^H\mathbf{A} + c\mathbf{I})\mathbf{x}^{(l)} = \mathbf{A}^H\mathbf{b}^{(l)}$. To overcome excessive memory requirement and computation time for inverse computation, the conjugate gradient (CG) method is employed as an iterative linear solver [4].

To validate the proposed method, real-time spiral balanced SSFP cardiac imaging [5] was conducted on a GE 1.5 T whole body scanner. A total of 100 image frames are reconstructed from the proposed method which uses the set of odd or even order interleaves alternately. Reference images corresponding to the same time points are generated using sliding window reconstruction. Regarding the zero assumption a circular window is used for each slice $I^{(l)}$ and its diameter changes in a step-wise way along frequency axis.

Results: Short axis images reconstructed using conventional sliding window and the proposed method are shown in Fig. 2. Motion artifacts, which appear as a background whirling, are reduced using the proposed method. Intensity profiles through the moderator band in the right ventricle (see cross-mark) are shown. The proposed method (solid line) produces sharper edges across the band, indicating reduced motion-blur.

Discussion: We propose an algebraic reconstruction method for reconstructing high temporal resolution images from dynamic spiral data undersampled in *k-t* space. In-vivo experiments using 2-fold undersampling show improved depiction of the moderator band in motion. We expect this method will improve dynamic spiral imaging especially in resolving rapid motion of fine structures such as valve leaflets.

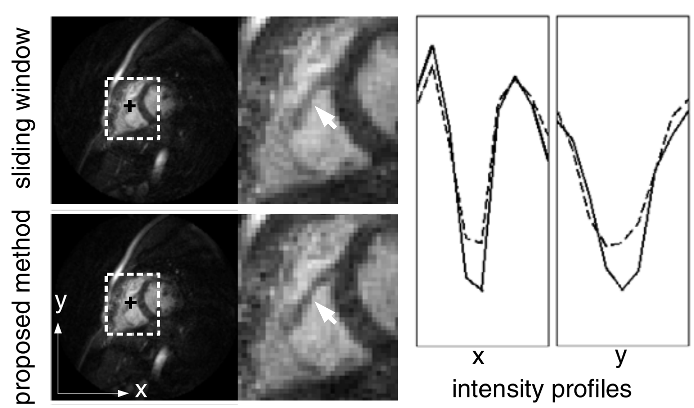

Fig. 2. Reconstructed short axis images and intensity profiles across moderator band

References

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