# A New Rician Noise Bias Correction

M. Nezamzadeh[1], I. Cameron[1,2]

[1]Physics, Carleton University, Ottawa, ON, Canada, [2]Radiology, The Ottawa Hospital, Ottawa, ON, Canada

**Introduction:** It is well known that pixel intensities of magnitude MR images are biased by Rician noise [1-4]. This is particularly noticeable for SNR ≤ 2. Rician noise bias corrections from the literature [2,3] work well for SNR > 2 but not for SNR < 2. Furthermore, the distribution of these "corrected" data points is far from Gaussian. This can have significant implications if the data is further processed using least squares procedures, which assume a Gaussian distribution of data points. In this work a new and improved procedure for Rician noise bias correction for small SNR is presented.

**Theory:** For SNR ≥ 3, the Rician PDF reduces to a Gaussian with a mean of $\overline{M} = A^2 + \sigma^2$, where A is the "actual" signal strength and σ is the standard deviation. This led Gudbjartsson and Patz [2] to propose that A could be estimated using

$$\tilde{A}_{GP} = \sqrt{\left|M_j^2 - \sigma^2\right|} \quad (1)$$

This works well for SNR ≥ 2 but not for SNR < 2 (See Table 1). The PDF for $\tilde{A}_{GP}$ is shown in Fig. 1 for SNR = 1.

A better estimator for A can be obtained from the same expression for $\overline{M}$ by using a binomial expansion. This expression is nice because it is a simple subtraction of a correction term which increases as $\overline{M}$ decreases (as it should) and the problematic square root is avoided. It also gives a better estimate of A while preserving the shape of the Rician PDF which is reasonably close to Gaussian even for small SNR. To improve the performance at low SNR we also made use of the fact that for SNR = 0, $\overline{M} = \sqrt{\frac{\pi}{2}}\sigma$ to get:

$$\tilde{A} \approx M_j - \overline{M}\left[\frac{1}{2}\left(\frac{n\sigma}{\overline{M}}\right)^2 + \frac{1}{8}\left(\frac{n\sigma}{\overline{M}}\right)^4 + \ldots\right] \quad \begin{cases} n = 1 & \text{for} \quad SNR \geq 1.5 \\ n = \sqrt{\frac{\pi}{2}} & \text{for} \quad SNR < 1.5 \end{cases} \quad (2)$$

$\overline{M}$ is not normally known but can be estimated as a nearest neighbour pixel average.

**Results:** A series of simulations were run for SNR = 0-3 with 500,000 samples with $\overline{M}$ estimated from the average of 9 points (i.e. nearest neighbour pixel averaging). The results were comparable to those of Gudbjartsson and Patz [2] for SNR ≥ 2 but superior for SNR < 2 (see Table 1). The PDFs for $M_j$, $\tilde{A}_{GP}$ and $\tilde{A}$ are shown in Fig 1 for SNR = 1. Our results are clearly superior for low SNR. In Fig 2 a simulated bi-exponential diffusion decay is shown along with the best fit obtained using least squares and the true decay curve calculated from the input parameters to the simulation. The corrected data points (using Eq. 2) are also shown along with the least squares fit to the corrected data. Clearly, using least squares procedures to obtain the parameters for the uncorrected diffusion decay leads to major errors for this example whereas the parameter values obtained for the corrected data are very close to the true values (see Table 2).

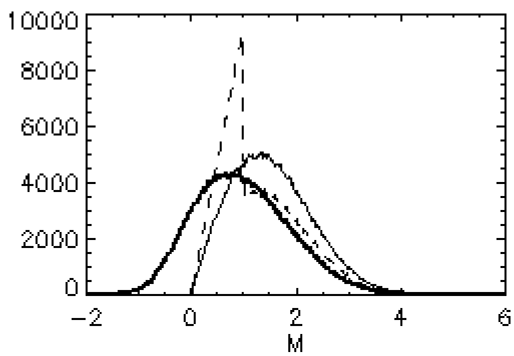


**Fig. 1** The PDFs for $M_j$ (thin solid line), $\tilde{A}_{GP}$ (dashed line) and $\tilde{A}$ (thick solid line)

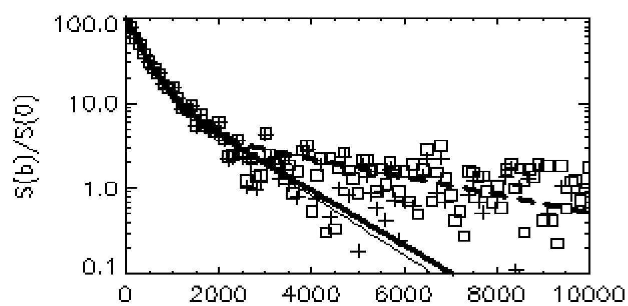


**Fig. 2:** Bi-exponential diffusion decay for Rician biased data (squares) and corrected data (+). The lines are the theoretical decay (thin solid line), the fit to the Rician biased data (dashed line) and the fit to the corrected data (thick solid line).

| SNR | $\overline{M}$ | $\tilde{A}_{GP}$ | $\tilde{A}$ |
|---|---|---|---|
| 0.0 | 1.253 | 1.034 | 0.222 |
| 0.5 | 1.328 | 1.100 | 0.464 |
| 1.0 | 1.547 | 1.299 | 0.989 |
| 1.5 | 1.876 | 1.619 | 1.549 |
| 2.0 | 2.273 | 2.030 | 2.035 |
| 2.5 | 2.710 | 2.495 | 2.517 |
| 3.0 | 3.174 | 2.991 | 3.011 |

**Table 1**

| | S(0) | $D_1$ (mm²/s) | $D_2$ (mm²/s) | f |
|---|---|---|---|---|
| **Input Parameters** | 100.0 | $3.00 \times 10^{-3}$ | $0.80 \times 10^{-3}$ | 0.80 |
| **Uncorrected Data** | 97.79 | $2.49 \times 10^{-3}$ | $0.24 \times 10^{-3}$ | 0.94 |
| **Corrected Data** | 99.08 | $2.95 \times 10^{-3}$ | $0.74 \times 10^{-3}$ | 0.81 |

**Table 2**

**Conclusions:** A new way of correcting Rician noise bias is proposed that performs better than existing methods when the SNR is small. It gives a better estimate for the true signal value and the PDF of the corrected data points is similar to a Gaussian which is an added benefit.

**References:** (1) Henkelman R.M.; Measurement of signal intensities in the presence of noise in MR images, Med. Phys., 12 (2), 1985. (2) Gudbjartsson H., Patz S.; The Rician distribution of noisy MRI data, MRM, 34,1995. (3) McGibney G., Smith M.R.; An unbiased signal-to-noise ratio measure for magnetic resonance images, Med. Phys., 20 (4), 1993. (4) Miller A.J., Joseph P.M.; The use of power images to perform quantitative analysis on low SNR MR images, Magn. Reson. Imaging, Vol. 11, 1051- 1056, 1993.